

Brief solutions to selected problems in homework 11

1. Section 5.3: Solutions, common mistakes and corrections:

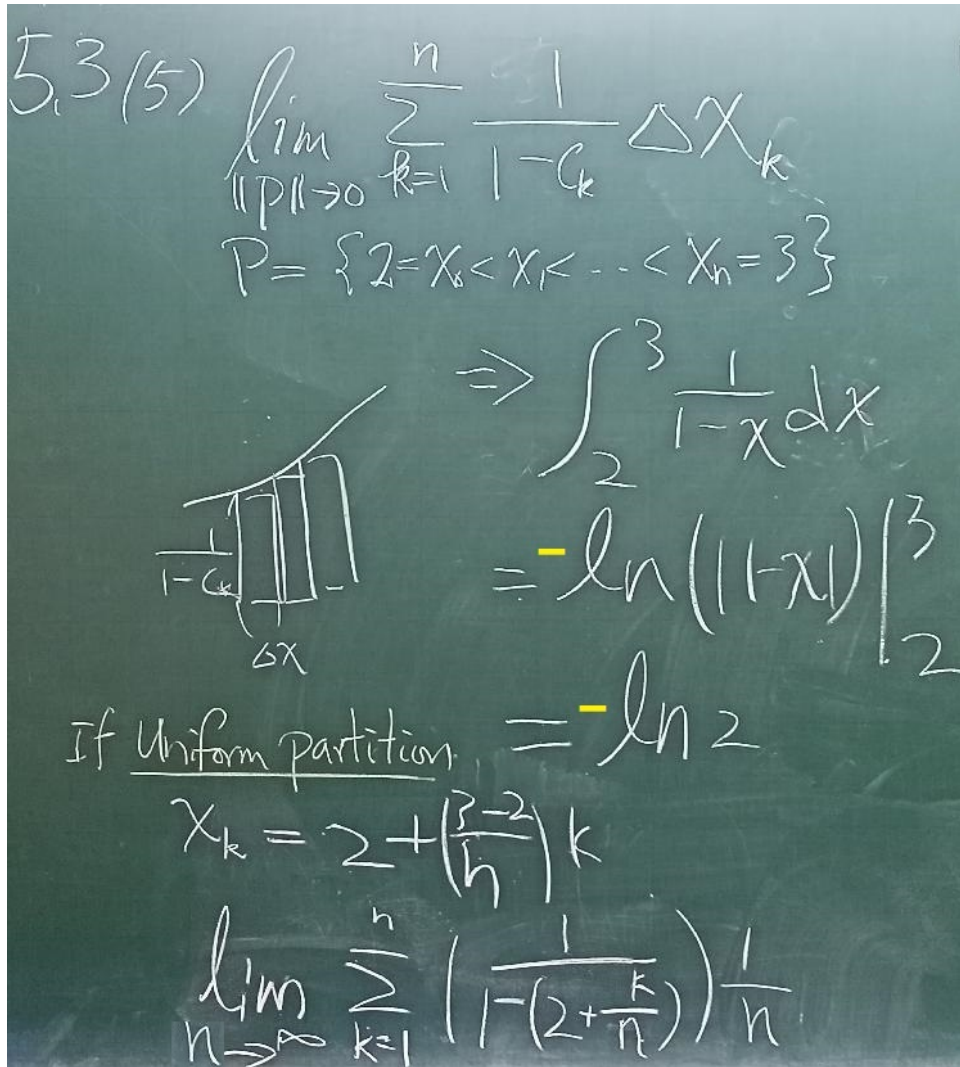


Figure 1: Solution to Section 5.3, problem 5

Correction for problem 5: $\int_2^3 \frac{1}{1-x} dx = - \int_2^3 \frac{1}{1-x} d(1-x) = -\ln(|1-x|) \Big|_2^3 = -\ln 2$

Problem 73: Since $\frac{1}{2} \leq \frac{1}{1+x^2} \leq 1$ on $[0, 1]$, we know from Table 5.6, item 6 that

$$\frac{1}{2} \leq \int_0^1 \frac{1}{1+x^2} \leq 1$$

2. Section 5.4: Solutions, common mistakes and corrections:

$$\begin{aligned} (15) \quad & \int_0^{\frac{\pi}{4}} \tan^2 x \, dx \quad (14) \\ &= \int_0^{\frac{\pi}{4}} (\sec^2 x - 1) \, dx \\ &= \tan x - x + C \Big|_0^{\frac{\pi}{4}} \\ &= 1 - \frac{\pi}{4} \quad \# \end{aligned}$$

Figure 2: Solution to Section 5.4, problem 15

$$\begin{aligned}
 (16) \int_0^{\frac{\pi}{6}} (\sec x + \tan x)^2 dx & \\
 &= \int_0^{\frac{\pi}{6}} \sec^2 x + 2 \sec x \tan x + \tan^2 x dx \\
 &= \int_0^{\frac{\pi}{6}} (2 \sec^2 x - 1 + 2 \sec x \tan x) dx \\
 &= 2 \tan x - x + 2 \sec x \Big|_0^{\frac{\pi}{6}} \\
 &= \frac{2}{\sqrt{3}} - \frac{\pi}{6} + 2 \left(\frac{2}{\sqrt{3}} - 1 \right)
 \end{aligned}$$

Figure 3: Solution to Section 5.4, problem 16

$$(34) \int_{-1}^0 \pi^{x-1} dx$$

$$x-1 = u$$

$$du = dx$$

$$\int_{-2}^{-1} \pi^u du$$

$$\Rightarrow \frac{\pi^u}{\ln \pi} \Big|_{-2}^{-1}$$

$$= \frac{1}{\ln \pi} \left(\frac{1}{\pi} - \frac{1}{\pi^2} \right)$$

$$\left(\frac{d}{dx} a^x = (\ln a) a^x \right)$$

Figure 4: Solution to Section 5.4, problem 34

$$\begin{aligned} (41) \quad & \frac{d}{dt} \int_0^{t^4} \sqrt{u} \, du \\ & \left(\frac{d}{dt^4} \int_0^{t^4} \sqrt{u} \, du \right) \cdot \frac{dt^4}{dt} \\ & = \sqrt{t^4} \cdot \frac{dt^4}{dt} \\ & = t^2 \cdot 4t^3 \\ & = 4t^5 \end{aligned}$$

Figure 5: Solution to Section 5.4, problem 41

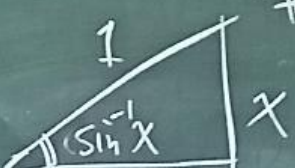
$$\begin{aligned}
 (55) \quad & \frac{d}{dx} \int_0^{\sin^{-1} x} \cos t \, dt \\
 & \left(\frac{d}{d \sin^{-1} x} \int_0^{\sin^{-1} x} \cos t \, dt \right) \cdot \frac{d \sin^{-1} x}{dx} \\
 & = \cos(\sin^{-1} x) \cdot \frac{d \sin^{-1} x}{dx} \\
 & = \cos(\sin^{-1} x) \cdot \frac{1}{\sqrt{1-x^2}} \\
 & = \frac{\sqrt{1-x^2}}{1} \cdot \frac{1}{\sqrt{1-x^2}} \\
 & = 1 \neq
 \end{aligned}$$


Figure 6: Solution to Section 5.4, problem 55

$$5.4(77) \int_1^x f(t) dt = x^2 - 2x + 1$$

$$f(x) = ?$$

$$\frac{d}{dx} \left(\int_1^x f(t) dt \right) = 2x - 2$$

$$f(x) = 2x - 2$$

Similar problem:

$$\int_1^x f(t) dt = x^2 - 2x + C$$

$$\Rightarrow f(x) = ? \quad C = ?$$

$$\Rightarrow f(x) = 2x - 2, \quad f(1) = 0 \Rightarrow C = 1$$

or $\int_a^x f(t) dt = x^2 - 2x + 1$

$$\Rightarrow f(x) = ? \quad a = ?$$

Figure 7: Solution to Section 5.4, problem 77

$$5.4(81) \quad g(x) = \int_0^x f(t) dt \quad f(1) = 0$$

$$f'(x) > 0$$

$$\frac{d}{dx} g(x) = f(x)$$

A: a. b. c (e)

$$d: \frac{f'(x) \quad + \quad - \quad +}{\quad \quad \quad | \quad \quad \quad}$$

$$e \quad f(x) \quad \nearrow \quad 0 \quad \nearrow$$

$$g'(x) \quad - \quad - \quad +$$

$\Rightarrow g$ has local min at $x=1$

$$(f, g): \quad g'' = f' > 0 \rightarrow \text{No inflection pt}$$

Figure 8: Solution to Section 5.4, problem 81

$$\begin{aligned}
 (84) \quad & \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}} \int_1^x \frac{1}{\sqrt{t}} dt \quad \left(\frac{\infty}{\infty} \right) \\
 &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}} \int_1^x t^{-\frac{1}{2}} dt \\
 &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}} \cdot 2t^{\frac{1}{2}} \Big|_1^x \\
 &= \lim_{x \rightarrow \infty} \frac{2\sqrt{x} - 2}{\sqrt{x}} \\
 &= 2 \neq \\
 & \text{L'Hôpital Rule:} \\
 &= \lim_{x \rightarrow \infty} \frac{\left(\int_1^x t^{-\frac{1}{2}} dt \right)'}{\left(x^{\frac{1}{2}} \right)'} = \lim_{x \rightarrow \infty} \frac{x^{-\frac{1}{2}}}{\frac{1}{2} x^{-\frac{1}{2}}} \\
 &= 2
 \end{aligned}$$

Figure 9: Solution to Section 5.4, problem 84

Remark: L'Hôpital's Rule can be used in cases where the antiderivative is not known, for example:

$$\lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}} \int_1^x \frac{1}{\sqrt{t} + \sin t} dx$$

3. Chapter 5, additional and advanced problems: Solutions, common mistakes and corrections:

(22)

$$\lim_{n \rightarrow \infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{1}{1+\frac{1}{n}} + \frac{1}{1+\frac{2}{n}} + \dots + \frac{1}{1+\frac{n}{n}} \right)$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{1}{1+\frac{i}{n}} \right) \frac{1}{n}$$

$$= \int_1^2 \frac{1}{x} dx = \ln|x| \Big|_1^2$$

Figure 10: Solution to Chapter 5, additional and advanced problems: problem 21