

Brief solutions to selected problems in homework 10

1. Section 4.6: Solutions, common mistakes and corrections:

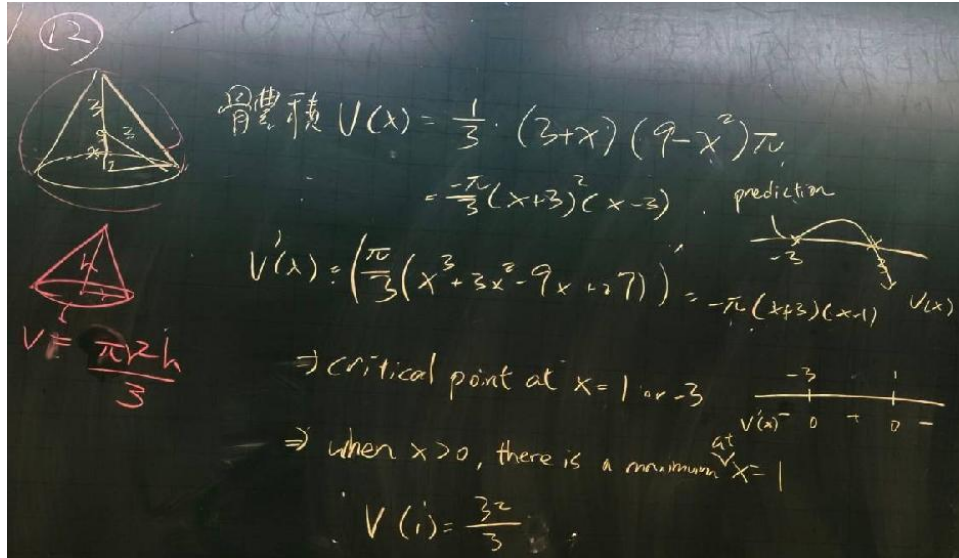


Figure 1: Solution to Section 4.6, problem 12

2. Section 4.8: Solutions, common mistakes and corrections:

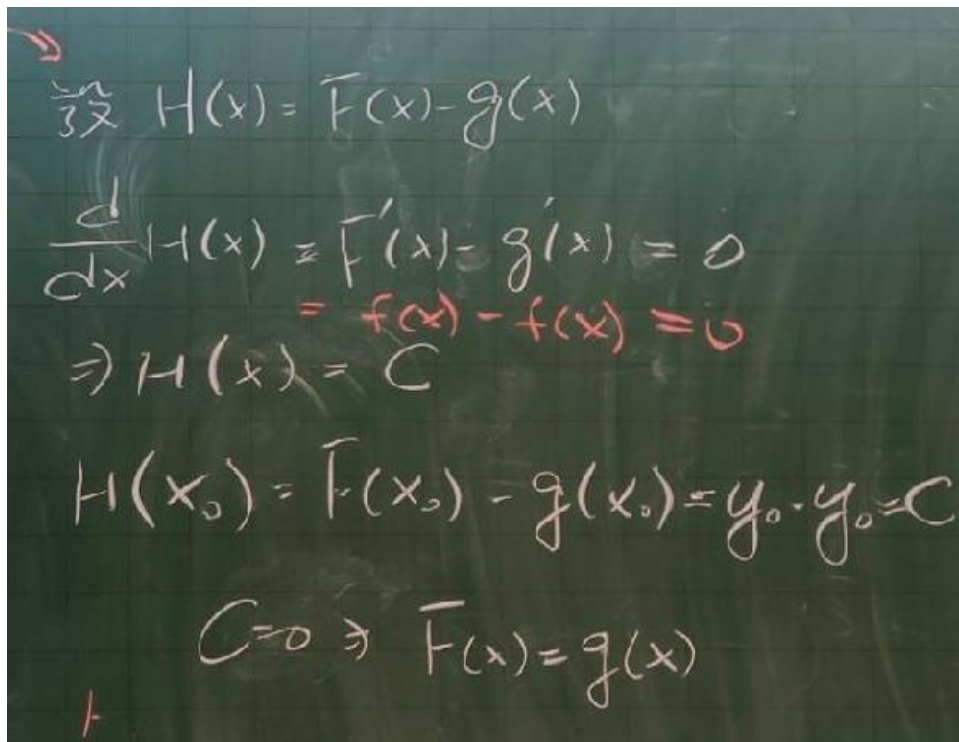


Figure 2: Solution to Section 4.8, problem 128

3. Section 5.2: Solutions, common mistakes and corrections:

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Let $x_0 < x_1 < \dots < x_{n-1} < x_n = 1$ be a partition of $[0, 1]$ with $x_i - x_{i-1} = \frac{1}{n}$ $i=1, \dots, n$

$$\sum_{i=1}^n \frac{1}{n} f(x_i)$$

$$= \sum_{i=1}^n \frac{1}{n} 2(x_i)^3$$

$$\neq \sum_{i=1}^n \frac{1}{n} 2 \left(\frac{x_i(x_i+1)}{2} \right)^2$$

$$= \sum_{i=1}^n \frac{1}{n} 2 \left(\frac{i}{n} \right)^3$$

$$= \frac{2}{n^4} \sum_{i=1}^n (i)^3 = \frac{2}{n^4} \left(\frac{(1+n)n}{2} \right)^2$$

$$= \frac{1}{2} \left(1 + \frac{2}{n} + \frac{1}{n^2} \right)$$

$$\Rightarrow \int_0^1 2x^3 dx$$

$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} f(x_i)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{2} \left(1 + \frac{2}{n} + \frac{1}{n^2} \right)$$

$$= \frac{1}{2}$$

Figure 3: Solution to Section 5.2, problem 45