

Brief solutions to selected problems in homework 09

1. Section 4.4: Solutions, common mistakes and corrections:

77

$$y' = x^{-\frac{2}{3}}(x-1)$$
$$y'' = x^{-\frac{2}{3}} - \frac{2}{3}x^{-\frac{5}{3}}(x-1)$$
$$= \frac{1}{3}x^{-\frac{2}{3}} + \frac{2}{3}x^{-\frac{5}{3}}$$
$$= \frac{1}{x^{\frac{2}{3}}}\left(\frac{1}{3}x + \frac{2}{3}\right)$$
$$f''(-2) = 0$$

Sign chart for $f''(x)$:

$f'(x)$	+	+	-	+
$f''(x)$	-	2	0	1

Graph of $f(x)$ showing a local maximum and a local minimum.

Figure 1: Solution to Section 4.4, problem 77

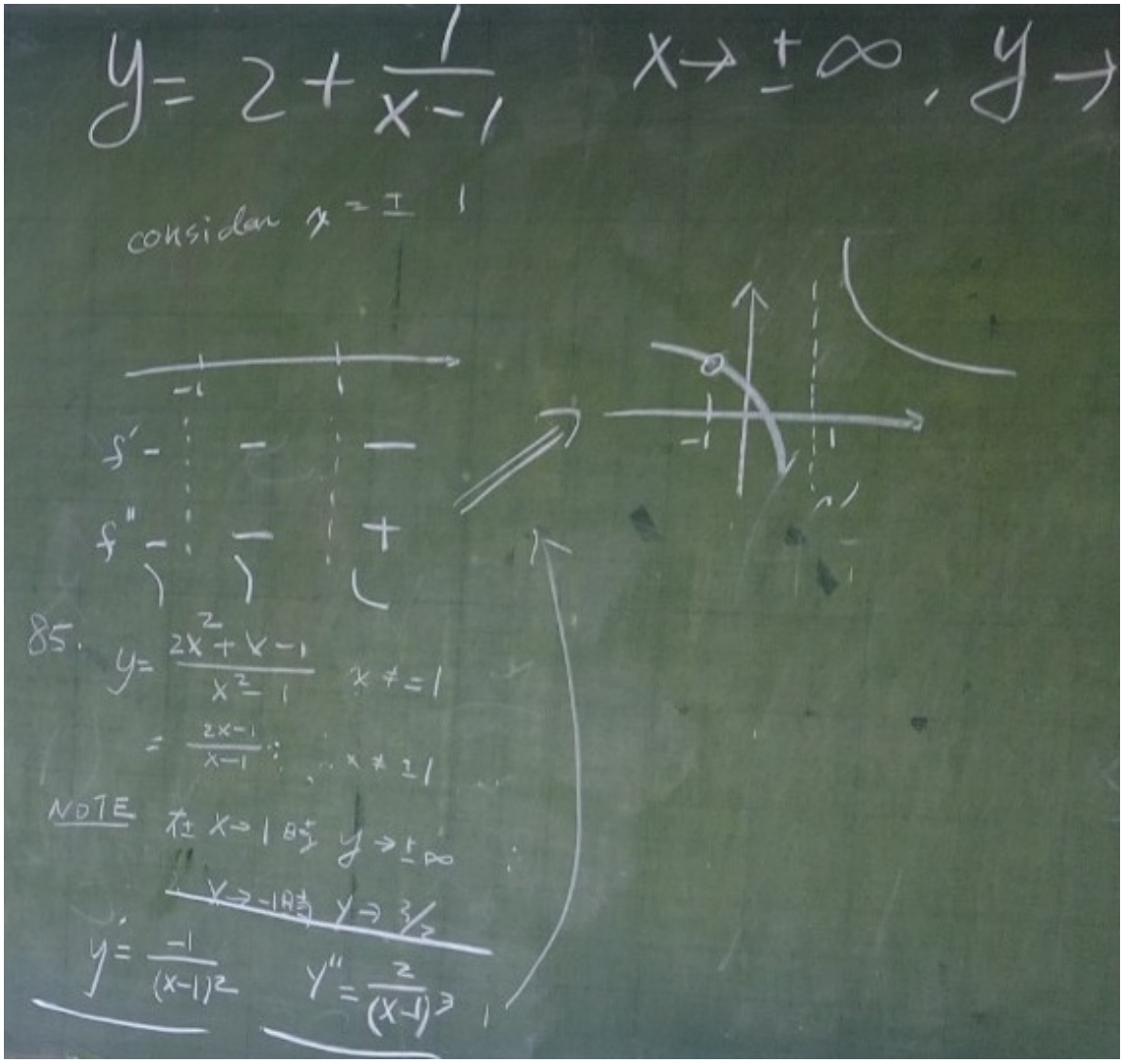


Figure 2: Solution to Section 4.4, problem 85

2. Section 4.5: Solutions, common mistakes and corrections:

$$\lim_{x \rightarrow 0^+} \frac{(\ln x)^2}{\ln(\sin x)} \quad \left(\frac{\infty}{-\infty} \right)$$

$$\text{L'H} \left\{ \begin{array}{l} \text{or} \\ \lim_{x \rightarrow 0^+} \frac{2 \ln x \cdot \frac{1}{x}}{\frac{\cos x}{\sin x}} \quad \left(\frac{-\infty}{\infty} \right) \xrightarrow{\text{L'H}} \lim_{x \rightarrow 0^+} \frac{2 \left(\frac{1 - \ln x}{x^2} \right)}{-\csc^2 x} \quad \left(\frac{\infty}{-\infty} \right) \\ = \text{繼續用 L'Hopital 只會更複雜} \\ \text{得不到結果} \end{array} \right.$$

$$\text{or} \\ \lim_{x \rightarrow 0^+} \left(\frac{\sin x}{x} \right) \left(\frac{2 \ln x}{\cos x} \right) \quad \left(1 \cdot \frac{-\infty}{1} = -\infty \right)$$

$$\lim_{x \rightarrow 0^+} \frac{(2 \ln x) \sin x}{x \cos x} \quad \left(\frac{-\infty \cdot 0}{0} \right) \quad \left. \begin{array}{l} \text{不知是否} \\ \frac{0}{0} \text{ 或 } \frac{\infty}{0} \\ \text{不能直接用 L'Hopital} \end{array} \right\}$$

需先確認 $\lim_{x \rightarrow 0^+} (\ln x) \sin x = 0$, 故為 $\frac{0}{0}$
 可再用 L'Hopital $= \lim_{x \rightarrow 0^+} \frac{2 \left(\frac{\sin x}{x} + \cos x \ln x \right)}{\cos x - x \sin x} \quad \left(\frac{\infty(-\infty)}{1-0} = -\infty \right)$

Figure 3: Solution to Section 4.5, problem 39

$$\lim_{x \rightarrow 0} \frac{\tan 2x + ax}{x^3} = -b$$

$$\lim_{x \rightarrow 0} \frac{2 \sec^2 2x + a}{3x^2} \quad \left(\begin{array}{l} \text{the lim exists} \\ \Rightarrow \text{must be "0/0"} \\ \Rightarrow a = -2 \end{array} \right)$$

$$\lim_{x \rightarrow 0} \frac{2 \sin^2 2x}{\cos^2 2x} = \frac{8}{3}$$

Figure 4: Solution to Section 4.5, problem 80

$$f(x) = e^{\ln f(x)} = e^{\frac{x \ln(1 + \frac{1}{x^2})}{1}}$$

84.1c)

Let $f(x) = (1 + \frac{1}{x^2})^x \Rightarrow \ln f(x) = \frac{1}{x} \ln(1 + \frac{1}{x^2})$

$$\Rightarrow \lim_{x \rightarrow \infty} x \ln(1 + \frac{1}{x^2}) = \lim_{x \rightarrow \infty} \frac{\ln(1 + \frac{1}{x^2})}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{1}{x^2}} \cdot \left(\frac{-2}{x^3}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \frac{2}{x} \left(\frac{1}{1 + \frac{1}{x^2}} \right) = 0$$

Figure 5: Solution to Section 4.5, problem 84(c)

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0} \frac{e^{-\frac{1}{x^2}}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{e^{\frac{1}{x^2}}} \xrightarrow{\text{L'Hopital's rule}} \lim_{x \rightarrow 0} \frac{-\frac{1}{x^2}}{e^{\frac{1}{x^2}} \cdot \left(-\frac{2}{x^3}\right)} = \lim_{x \rightarrow 0} \frac{x}{e^{\frac{1}{x^2}} \cdot 2} = 0$$

L'Hopital's rule

Figure 6: Solution to Section 4.5, problem 88