

Brief solutions to selected problems in homework 06

1. Section 3.8: Solutions, common mistakes and corrections:

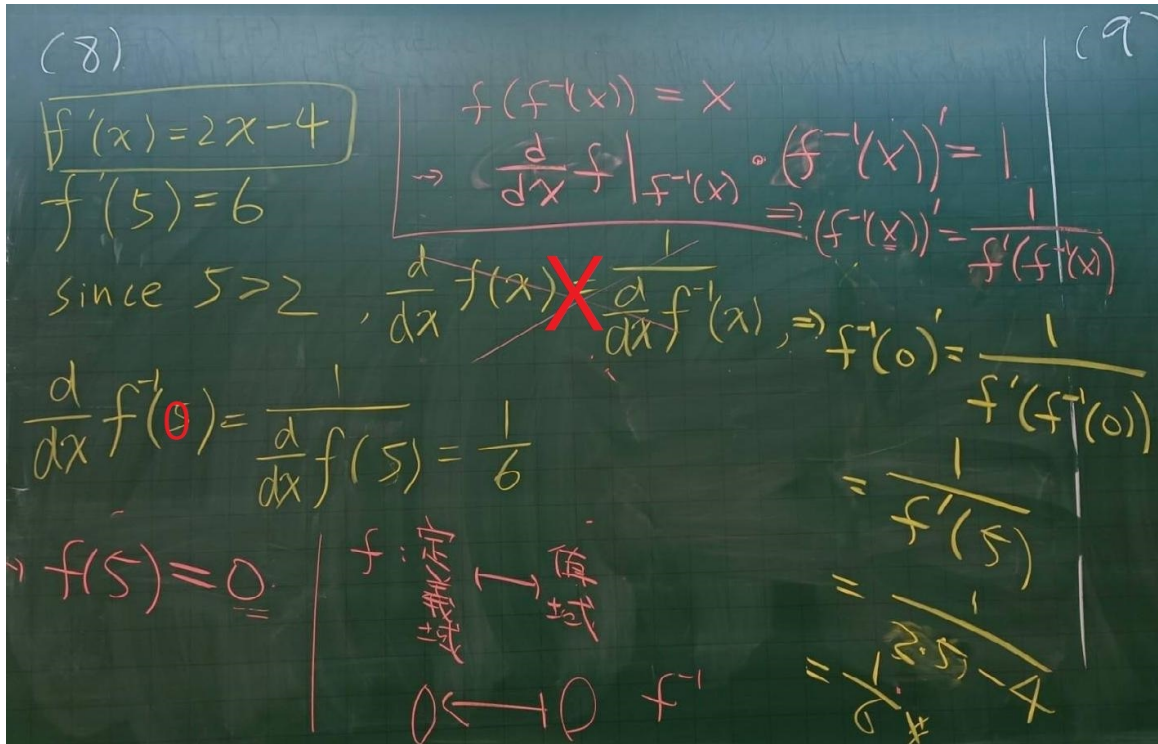


Figure 1: Solution to Section 3.8, problem 8 and some corrections

$$\begin{aligned}
 y &= (\sin x)^x \quad \text{Q} \\
 \ln y &= x \ln(\sin x) \\
 \frac{y'}{y} &= \ln(\sin x) + x \cdot \frac{\cos x}{\sin x} \\
 \frac{y'}{y} &= \ln(\sin x) + x \cot x \\
 y' &= y (\ln(\sin x) + x \cot x)
 \end{aligned}$$

Figure 2: Solution to Section 3.8, problem 93. Alternative method: start with $y = (e^{\ln \sin x})^x = e^{x \ln \sin x}$

$$\begin{aligned}
 95 \quad y &= x^{\ln x}, \quad x > 0 \\
 \ln y &= \ln x^{\ln x} = (\ln x)^2 \\
 \frac{y'}{y} &= \left(\frac{x'}{x} + \ln x \right) \times 2 \\
 \frac{y'}{y} &= \frac{2 \ln x}{x}, \quad y' = 2x^{\ln x} \times \frac{\ln x}{x}
 \end{aligned}$$

Figure 3: Solution to Section 3.8, problem 95. Alternative method: start with $y = (e^{\ln x})^x = e^{(\ln x)^2}$

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Show that $\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x \quad \forall x > 0$

pf.

$$\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^{\left(x \lim_{n \rightarrow \infty} \frac{\ln\left(1 + \frac{x}{n}\right)}{\frac{x}{n}}\right)} = e^x \lim_{t \rightarrow 0^+} \frac{\ln(1+t) - \ln(1)}{t}$$

$$= \lim_{n \rightarrow \infty} e^{n \ln\left(1 + \frac{x}{n}\right)} = e^x \lim_{n \rightarrow \infty} \frac{\ln\left(1 + \frac{x}{n}\right)}{\frac{x}{n}}$$

Since e^x is cont. on \mathbb{R}

$$= e^x \left(\lim_{n \rightarrow \infty} n \ln\left(1 + \frac{x}{n}\right)\right) = e^x \lim_{t \rightarrow 0^+} \frac{\ln(1+t)}{t}$$

$$\begin{aligned} &= \lim_{m \rightarrow 1} \frac{\ln(m)}{m-1} \\ &= \lim_{m \rightarrow 1} \frac{1}{m} \end{aligned}$$

Figure 4: Solution to Section 3.8, problem 98