Calculus I, Fall 2023 (Thomas' Calculus Early Transcendentals 13ed), http://www.math.nthu.edu.tw/~wangwc/

## Brief solutions to selected problems in homework 05

1. Section 3.5: Solutions, common mistakes and corrections:

## Problem 58:

Continuity: We have  $\lim_{x\to 0^+} g(x) = g(0) = 1$  and  $\lim_{x\to 0^-} g(x) = b$ . Therefore  $\lim_{x\to 0^+} g(x) = \lim_{x\to 0^-} g(x) = g(0)$  implies g is continuous at x = 0 if and only if b = 1. Differentiability:  $\lim_{x\to 0^+} \frac{g(x) - g(0)}{x - 0} = 0$ ,  $\lim_{x\to 0^-} \frac{g(x) - g(0)}{x - 0} = 1$  for any  $b \in \mathbb{R}$ , therefore

 $\lim_{x \to 0} \frac{g(x) - g(0)}{x - 0} \text{ does not exist. } g \text{ is not differentiable at } x = 0 \text{ for any } b \in \mathbb{R}.$ 

2. Problem 2:

$$\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \frac{x\cos x - \sin x}{x^2} = \frac{\cos x - \frac{\sin x}{x}}{x}$$
From the inequality (page 104 of the textbody)

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$$1 > \frac{\sin x}{x} > \cos x$$
, on  $0 < x < \frac{\pi}{2}$ ,

which also holds for  $0 > x > \frac{-\pi}{2}$  (since both  $\frac{\sin x}{x}$  and  $\cos x$  are even functions), we see that if  $0 < |x| < \frac{\pi}{2}$ , then

$$\left|\frac{d}{dx}\left(\frac{\sin x}{x}\right)\right| = \left|\frac{\cos x - \frac{\sin x}{x}}{x}\right| = \frac{\frac{\sin x}{x} - \cos x}{|x|} < \frac{1 - \cos x}{|x|} = \frac{2\sin^2 \frac{x}{2}}{|x|} < \frac{2\left(\frac{x}{2}\right)^2}{|x|} = \frac{|x|}{2}$$

Overall, we have

$$0 \le \left| \frac{d}{dx} \left( \frac{\sin x}{x} \right) \right| < \frac{|x|}{2}, \quad \text{on } 0 < |x| < \frac{\pi}{2}$$

Since  $\lim_{x\to 0} \frac{|x|}{2} = 0$ , it follows from the Sandwich Theorem that  $\lim_{x\to 0} \left| \frac{d}{dx} \left( \frac{\sin x}{x} \right) \right| = 0$ , and therefore  $\lim_{x\to 0} \frac{d}{dx} \left( \frac{\sin x}{x} \right) = 0$ .

3. Problem 4:

Ans: 
$$= f'(g(2)) \cdot g'(2) = f'(3) \cdot g'(2) = 0.4$$



4. Section 3.7: Solutions, common mistakes and corrections:

Figure 1: Solution to Section 3.7, problem 48

Suppose they meet at (a,b)  $a^{2}tb^{2}=y$   $a^{2}=$ 9 = X  $\begin{array}{c} (D = 1) \\ 2\alpha + 2b \vartheta_{1}(\alpha) = 0 \\ \Rightarrow \\ y_{1}(\alpha) = -\frac{\alpha}{b} \\ \Rightarrow \\ y_{2}'(\alpha) = -\frac{\alpha}{3b} \\ \Rightarrow \\ y_{1}'(\alpha) \vartheta_{1}'(\alpha) - -\frac{\alpha}{b} \\ \Rightarrow \\ \end{array}$ 2X+24,4,=0 01 36

Figure 2: Solution to Section 3.7, problem 51(a)