Brief solutions to selected problems in homework 04

1. Section 3.2: Solutions, common mistakes and corrections:

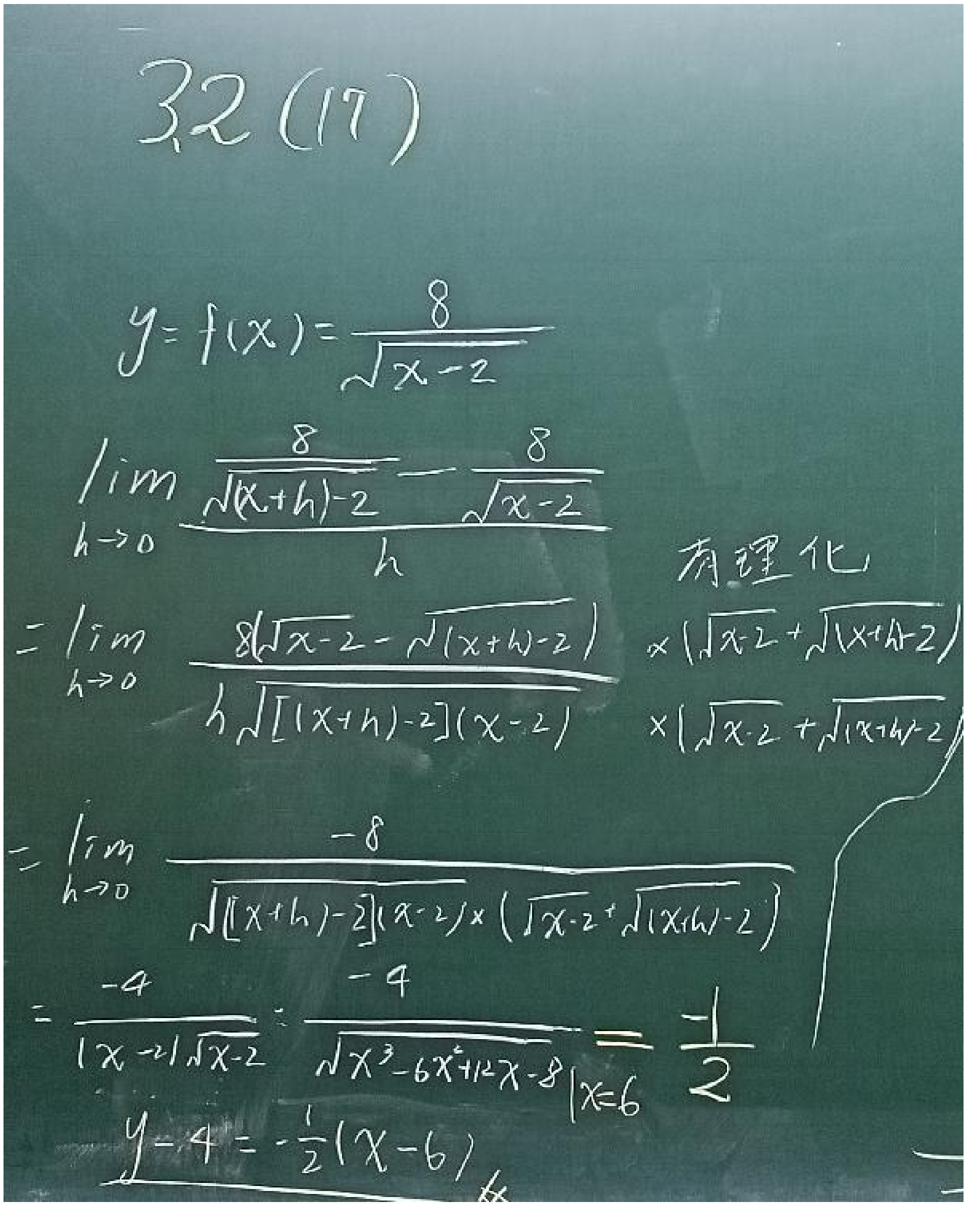


Figure 1: Solution to Section 3.2, problem 17

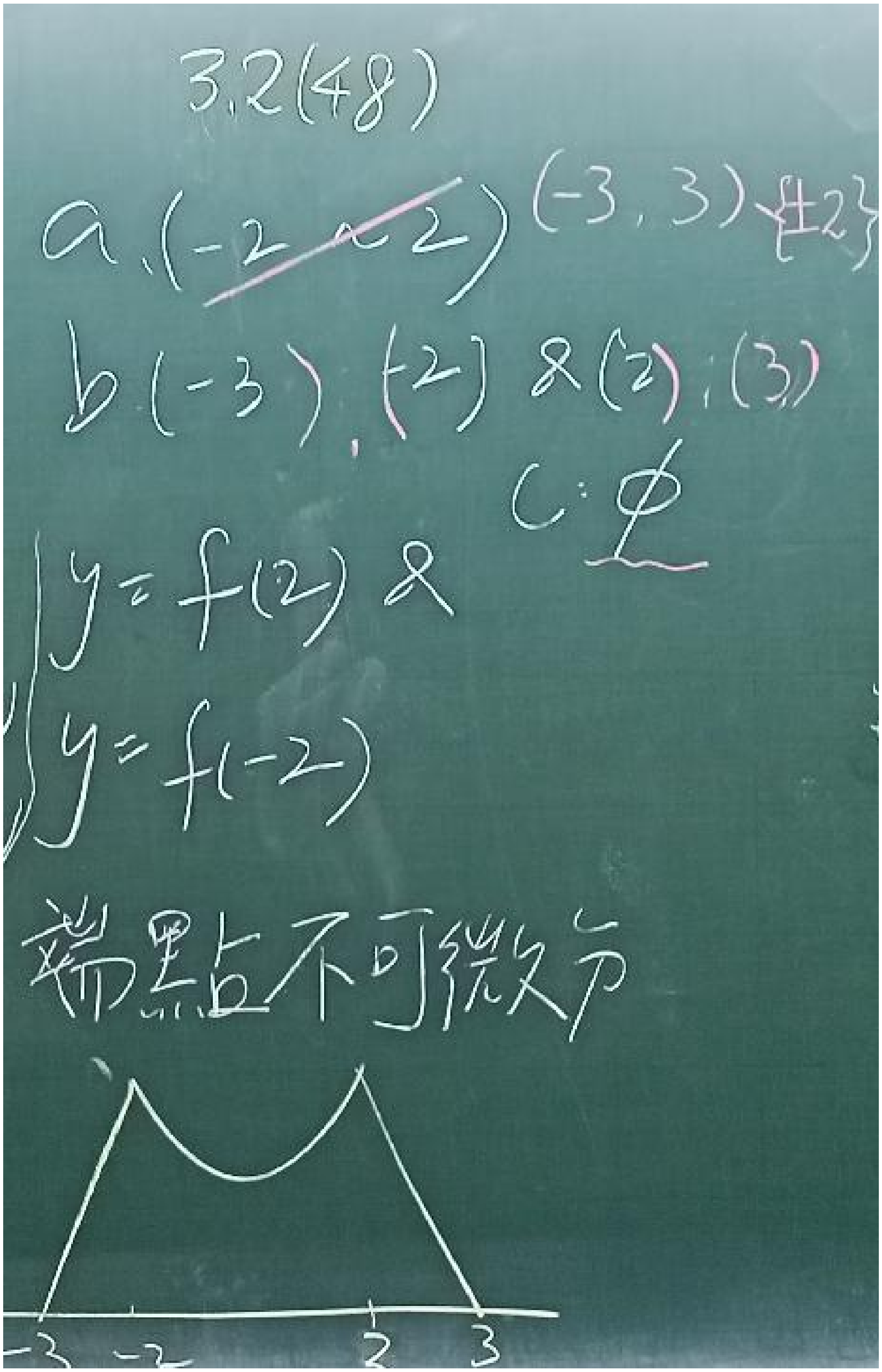


Figure 2: Solution to Section 3.2, problem 48

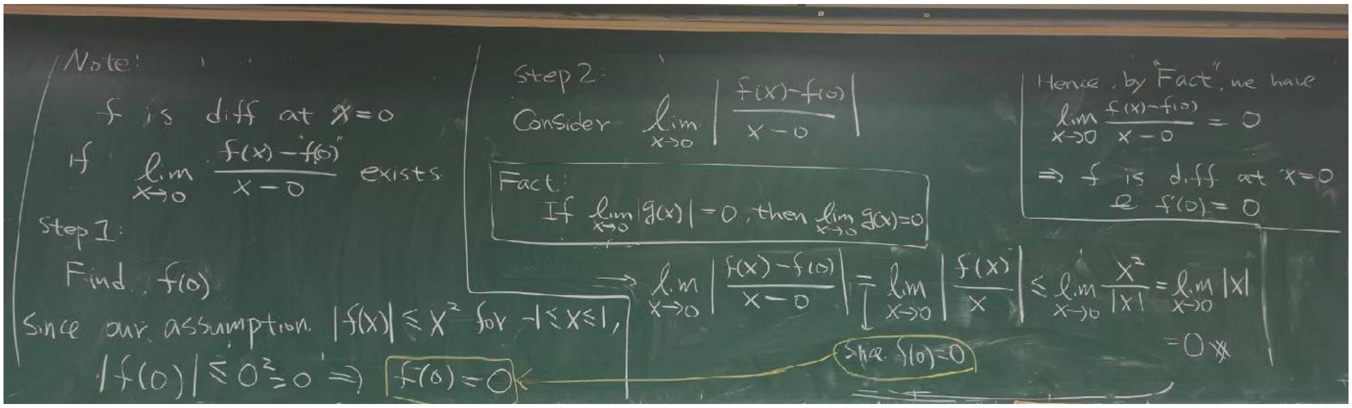


Figure 3: Solution to Section 3.2, problem 58(a). Step 1: show that $f(0) = 0$. Step 2: show that $f'(0) = 0$ as in problem 58(b)

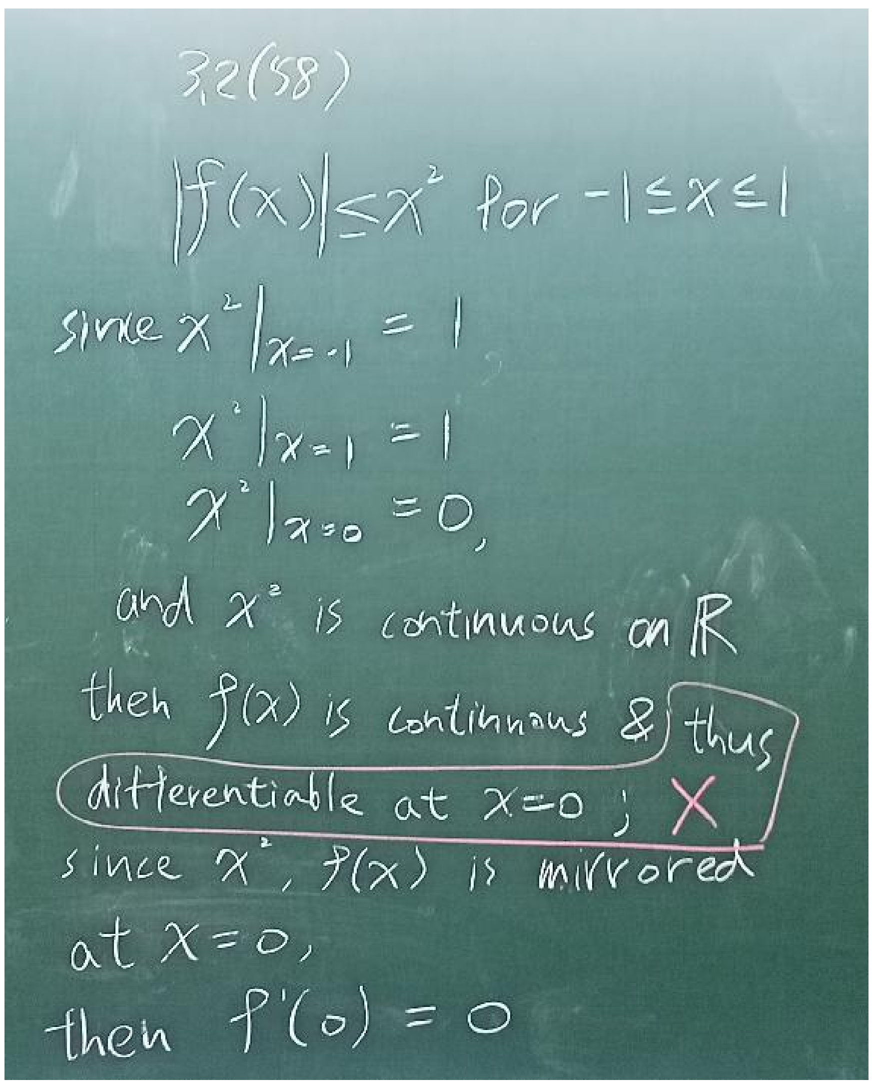


Figure 4: Common mistakes to Section 3.2, problem 58(a)

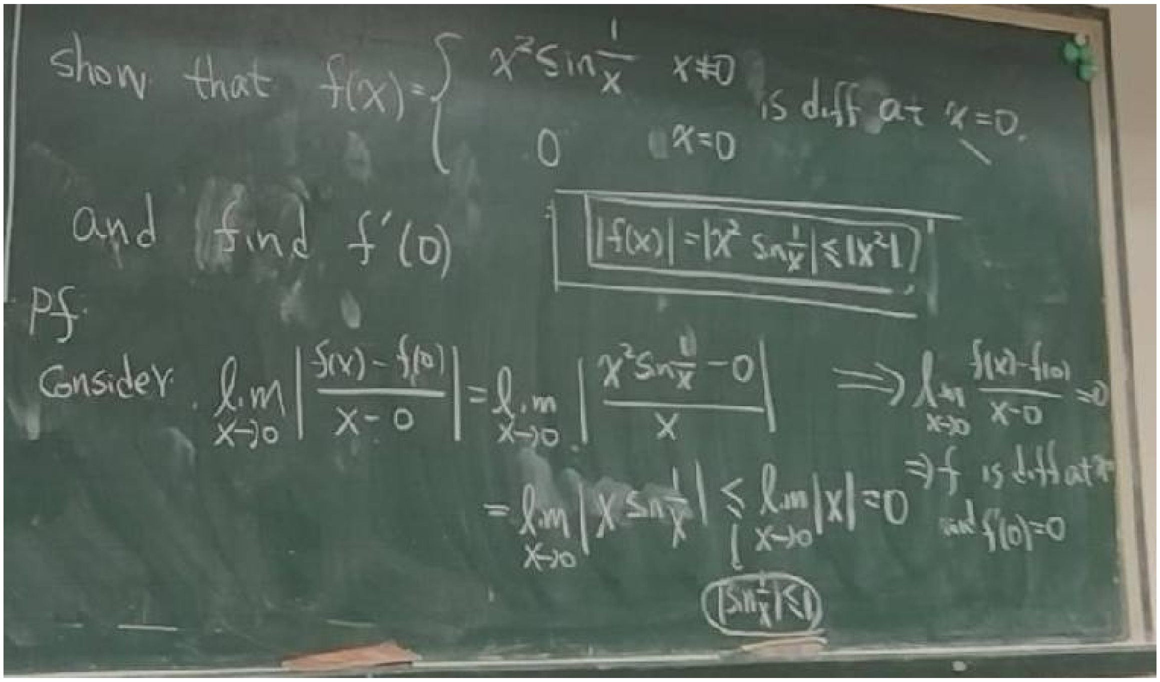


Figure 5: Solution to Section 3.2, problem 58(b)

2. Section 3.3: Solutions, common mistakes and corrections:

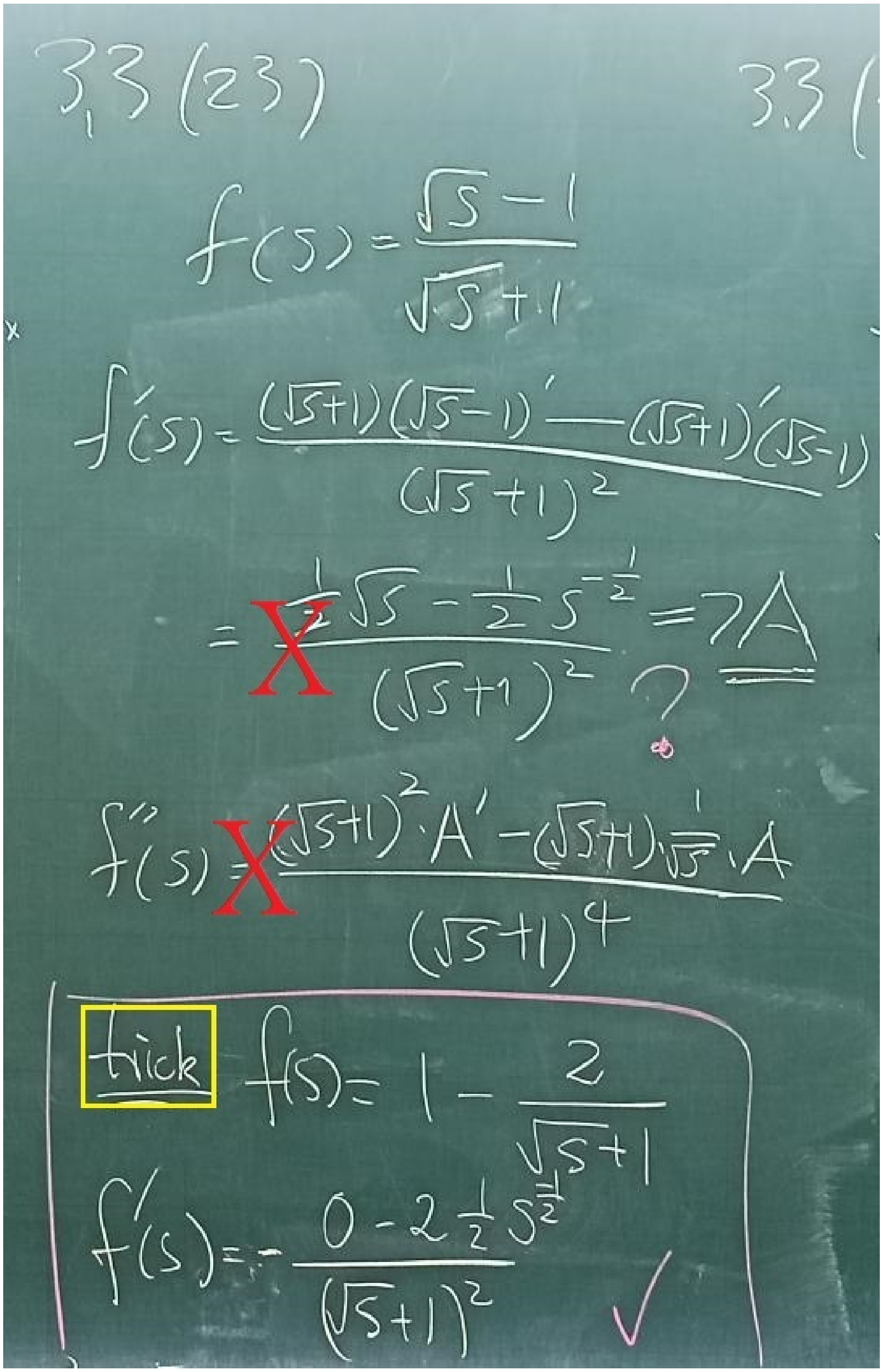


Figure 6: A trick for Section 3.3, problem 23

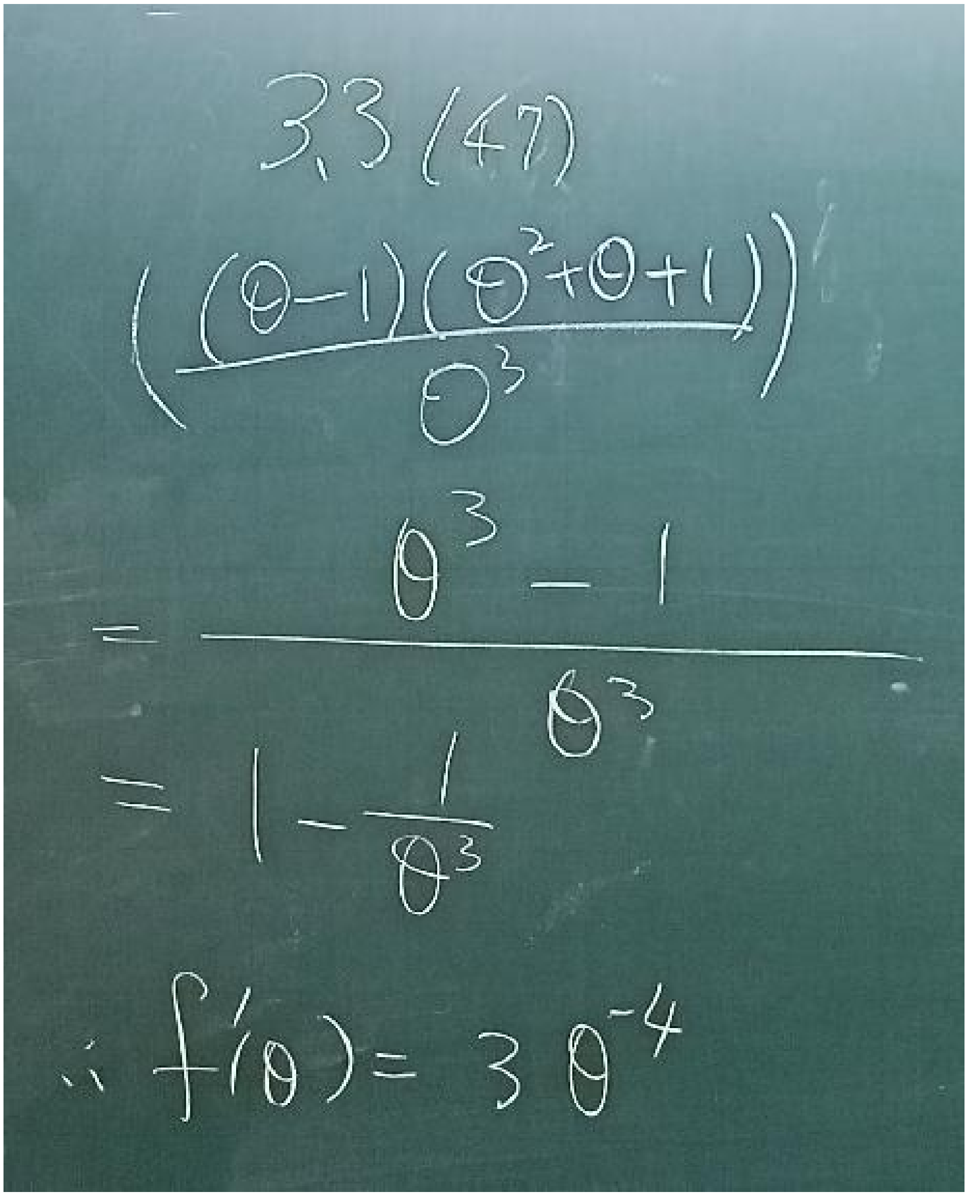


Figure 7: A trick for Section 3.3, problem 47

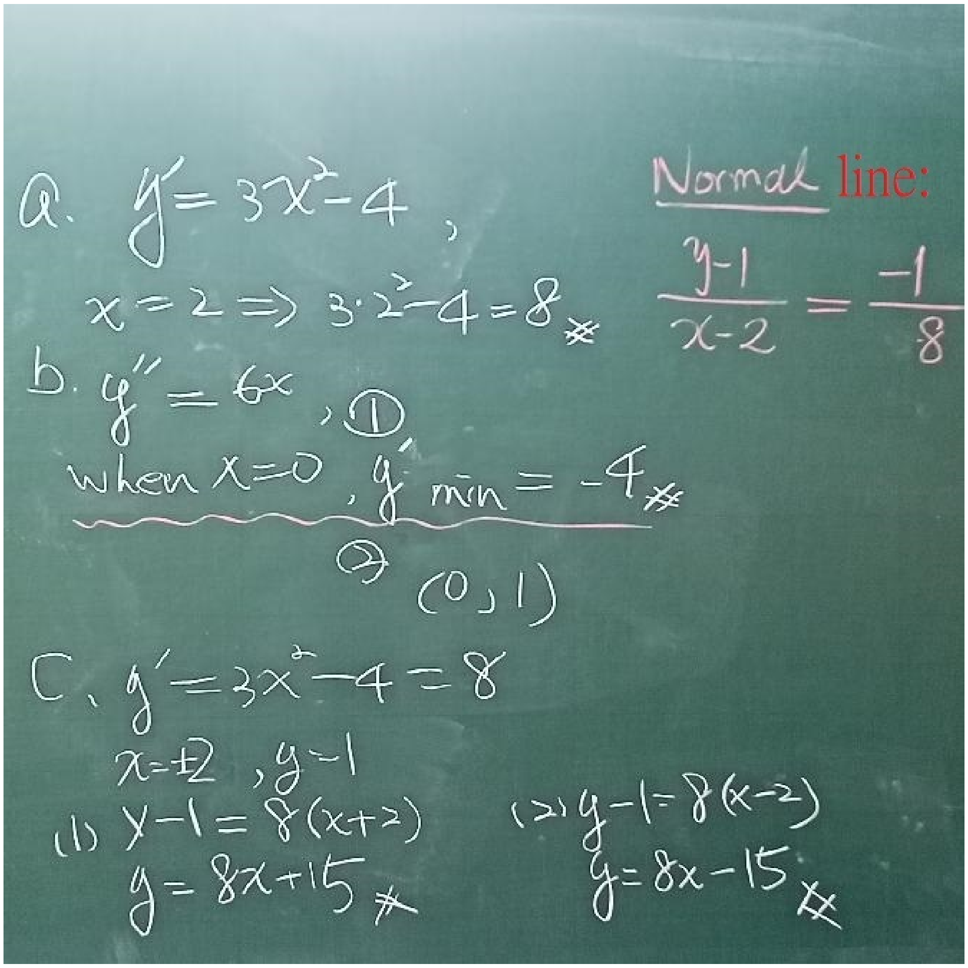


Figure 8: Solution to Section 3.3, problem 55

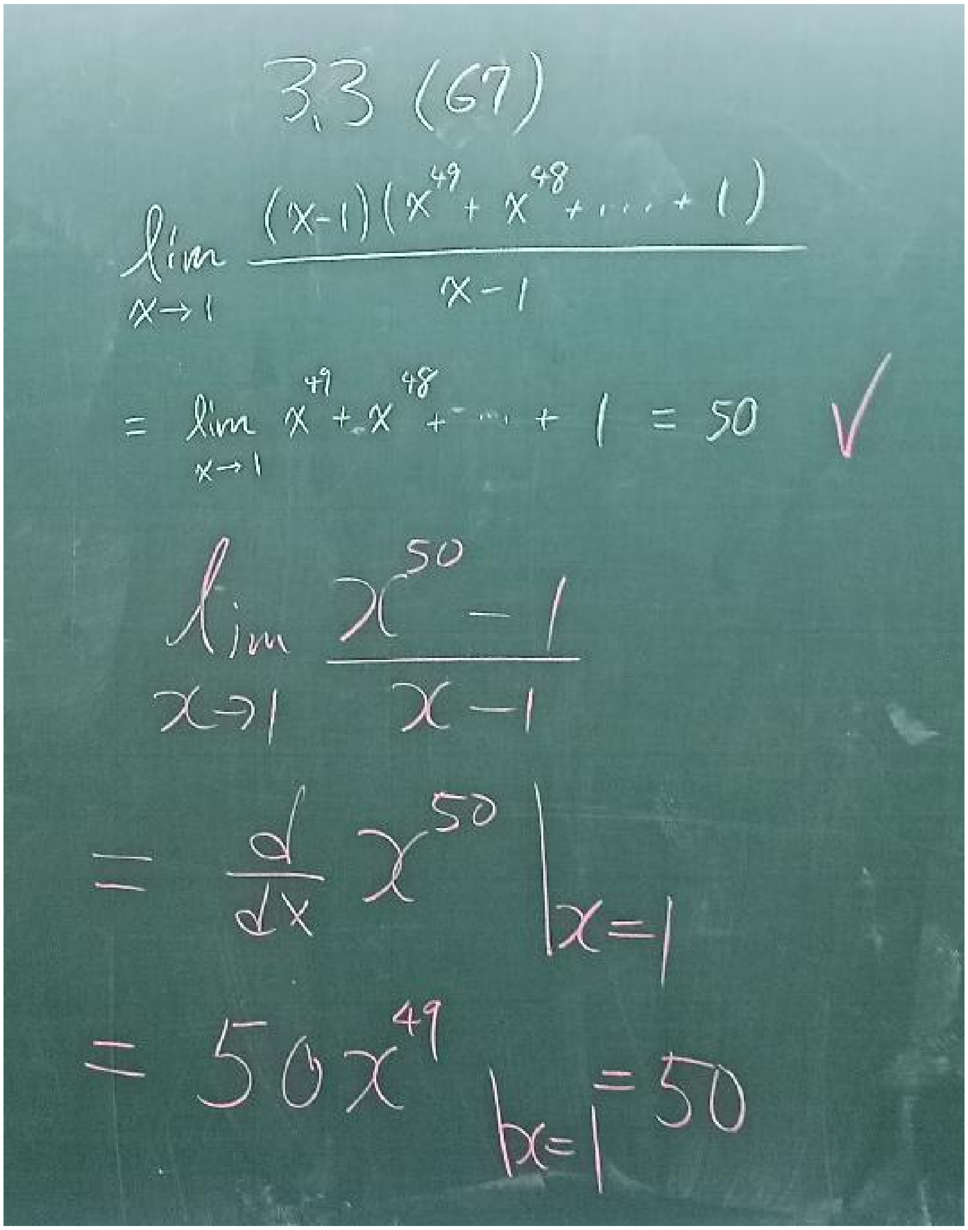


Figure 9: A trick for Section 3.3, problem 67

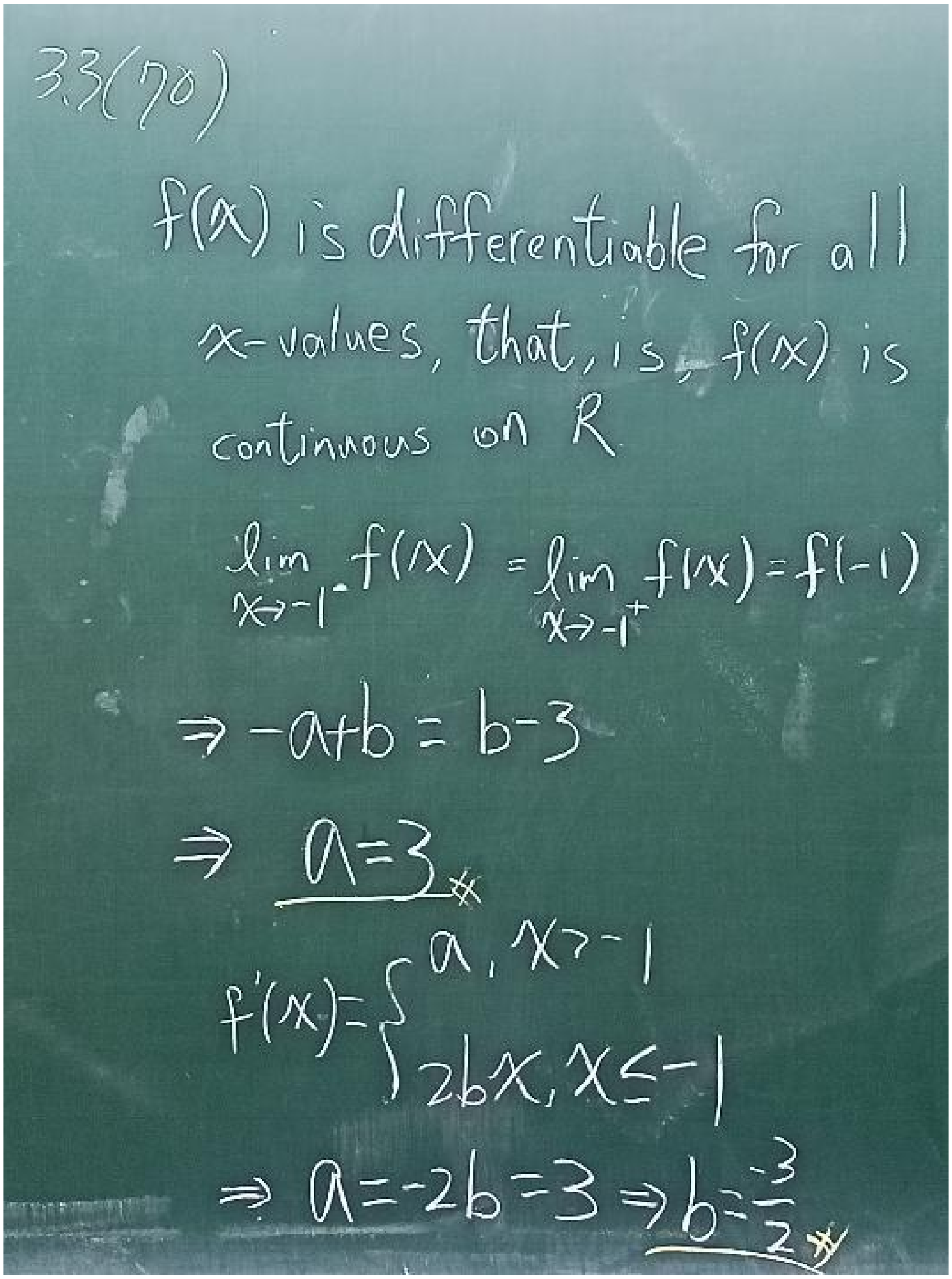
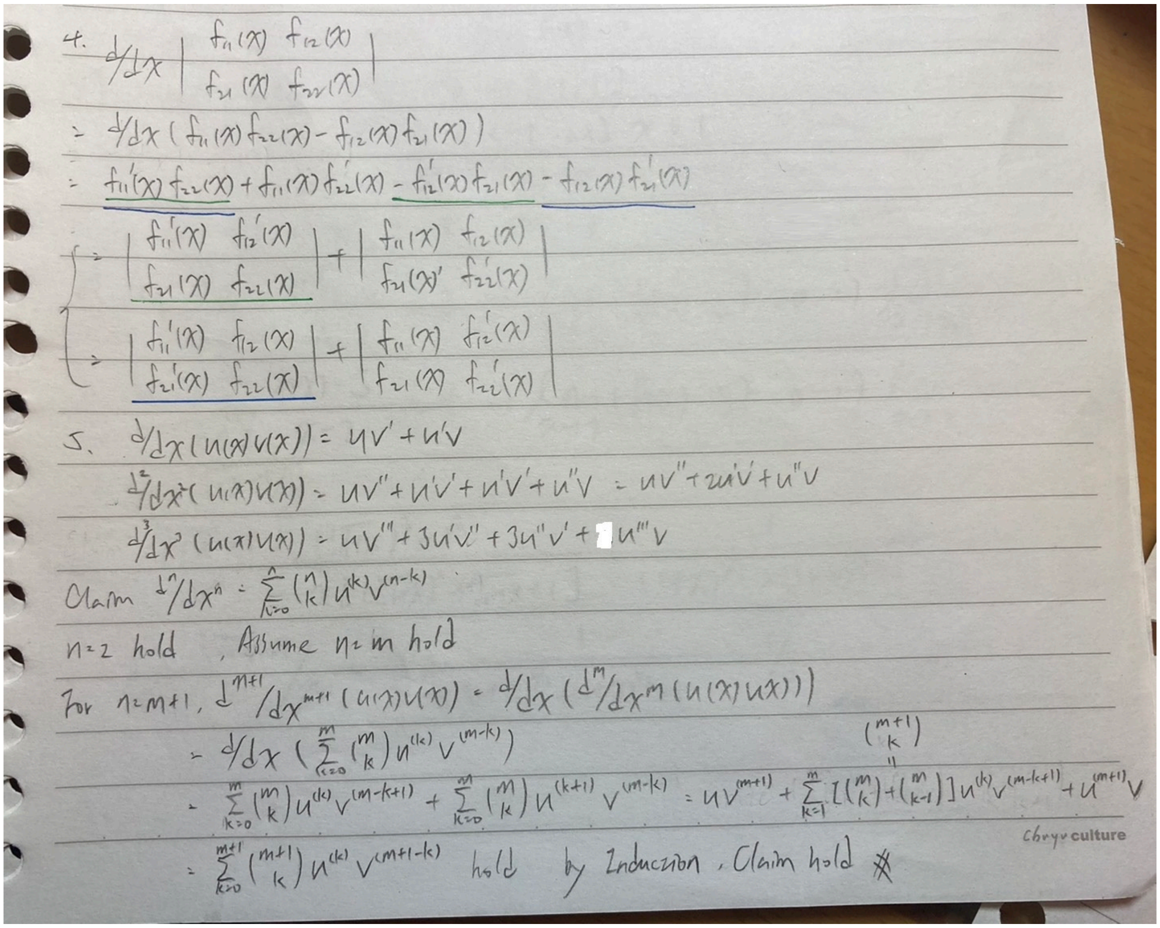


Figure 10: Solution to Section 3.3, problem 70

3. Solution to Homework 04, problem 4,5:



4.
$$\frac{d}{dx}\begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix}$$
$$= \frac{d}{dx}\left(f_{11}(x)f_{22}(x) - f_{12}(x)f_{21}(x)\right)$$
$$= f_{11}'(x)f_{22}(x) + f_{11}(x)f_{22}'(x) - f_{12}'(x)f_{21}(x) - f_{12}(x)f_{21}'(x)$$
$$= \begin{vmatrix} f_{11}'(x) & f_{12}'(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f_{21}(x)' & f_{22}'(x) \end{vmatrix}$$
$$= \begin{vmatrix} f_{11}'(x) & f_{12}(x) \\ f_{21}'(x) & f_{22}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}'(x) \\ f_{21}(x) & f_{22}'(x) \end{vmatrix}$$

5.
$$\frac{d}{dx}(u(x)v(x)) = uv' + u'v$$
$$\frac{d^2}{dx^2}(u(x)v(x)) = uv'' + u'v' + u'v' + u''v = uv'' + 2u'v' + u''v$$
$$\frac{d^3}{dx^3}(u(x)v(x)) = uv''' + 3u'v'' + 3u''v' + \;\; u'''v$$

Claim $\frac{d^n}{dx^n} = \sum_{k=0}^{n}\binom{n}{k}u^{(k)}v^{(n-k)}$

$n=2$ hold, Assume $n=m$ hold

For $n=m+1$, $\frac{d^{m+1}}{dx^{m+1}}(u(x)v(x)) = \frac{d}{dx}\left(\frac{d^m}{dx^m}(u(x)v(x))\right)$
$$= \frac{d}{dx}\left(\sum_{k=0}^{m}\binom{m}{k}u^{(k)}v^{(m-k)}\right)$$
$$= \sum_{k=0}^{m}\binom{m}{k}u^{(k)}v^{(m-k+1)} + \sum_{k=0}^{m}\binom{m}{k}u^{(k+1)}v^{(m-k)} = uv^{(m+1)} + \sum_{k=1}^{m}\left[\binom{m}{k}+\binom{m}{k-1}\right]u^{(k)}v^{(m-k+1)} + u^{(m+1)}v$$
$$\left[\binom{m}{k}+\binom{m}{k-1}\right] = \binom{m+1}{k}$$
$$= \sum_{k=0}^{m+1}\binom{m+1}{k}u^{(k)}v^{(m+1-k)}$$ hold by Induction, Claim hold ※

Figure 11: Solution to Homework 04, problem 4,5