

Brief solutions to selected problems in homework 04

1. Section 3.2: Solutions, common mistakes and corrections:

3.2 (17)

$$y = f(x) = \frac{8}{\sqrt{x-2}}$$

$$\lim_{h \rightarrow 0} \frac{\frac{8}{\sqrt{(x+h)-2}} - \frac{8}{\sqrt{x-2}}}{h}$$

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$$= \lim_{h \rightarrow 0} \frac{8(\sqrt{x-2} - \sqrt{(x+h)-2})}{h \sqrt{[(x+h)-2](x-2)}} \times \frac{(\sqrt{x-2} + \sqrt{(x+h)-2})}{(\sqrt{x-2} + \sqrt{(x+h)-2})}$$

$$= \lim_{h \rightarrow 0} \frac{-8}{\sqrt{[(x+h)-2](x-2)} (\sqrt{x-2} + \sqrt{(x+h)-2})}$$

$$= \frac{-4}{(x-2)\sqrt{x-2}} = \frac{-4}{\sqrt{x^3 - 6x^2 + 12x - 8}} \Big|_{x=6} = \frac{-1}{2}$$

$$y - 4 = -\frac{1}{2}(x-6)$$

Figure 1: Solution to Section 3.2, problem 17

3.2(48)

$$a. (-2, 2) \cup (-3, 3) \setminus \{2\}$$

$$b. (-3, 2) \cup (2, 3)$$

$$c. \emptyset$$

$$y = f(2) \cup$$

$$y = f(-2)$$

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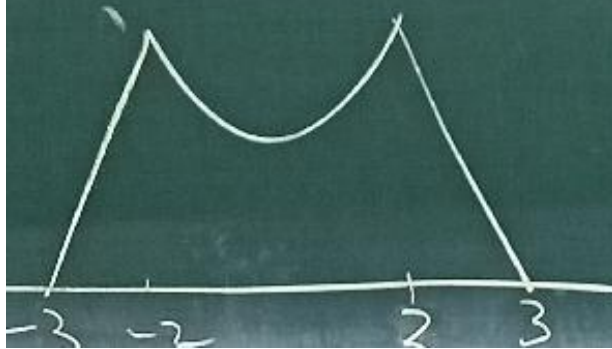


Figure 2: Solution to Section 3.2, problem 48

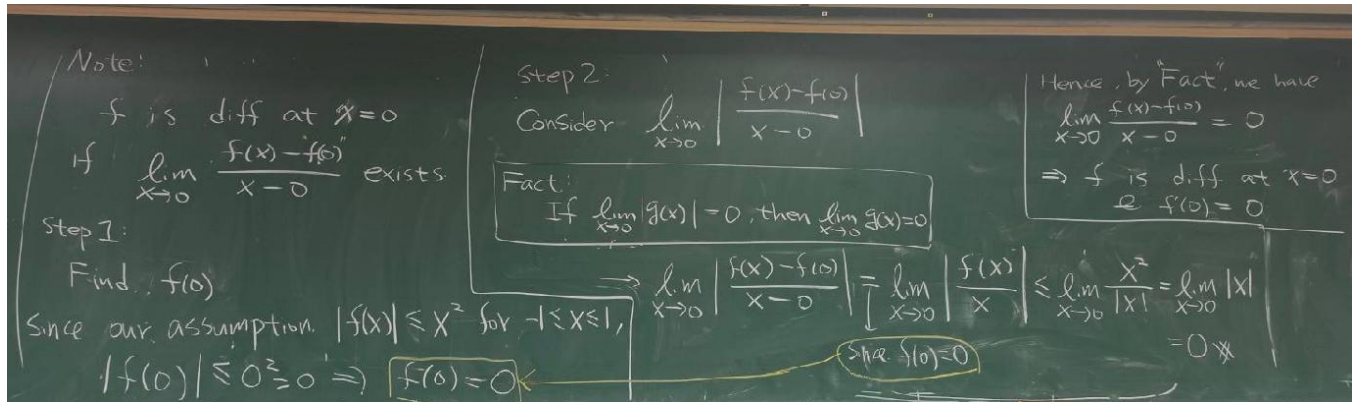


Figure 3: Solution to Section 3.2, problem 58(a). Step 1: show that $f(0) = 0$. Step 2: show that $f'(0) = 0$ as in problem 58(b)

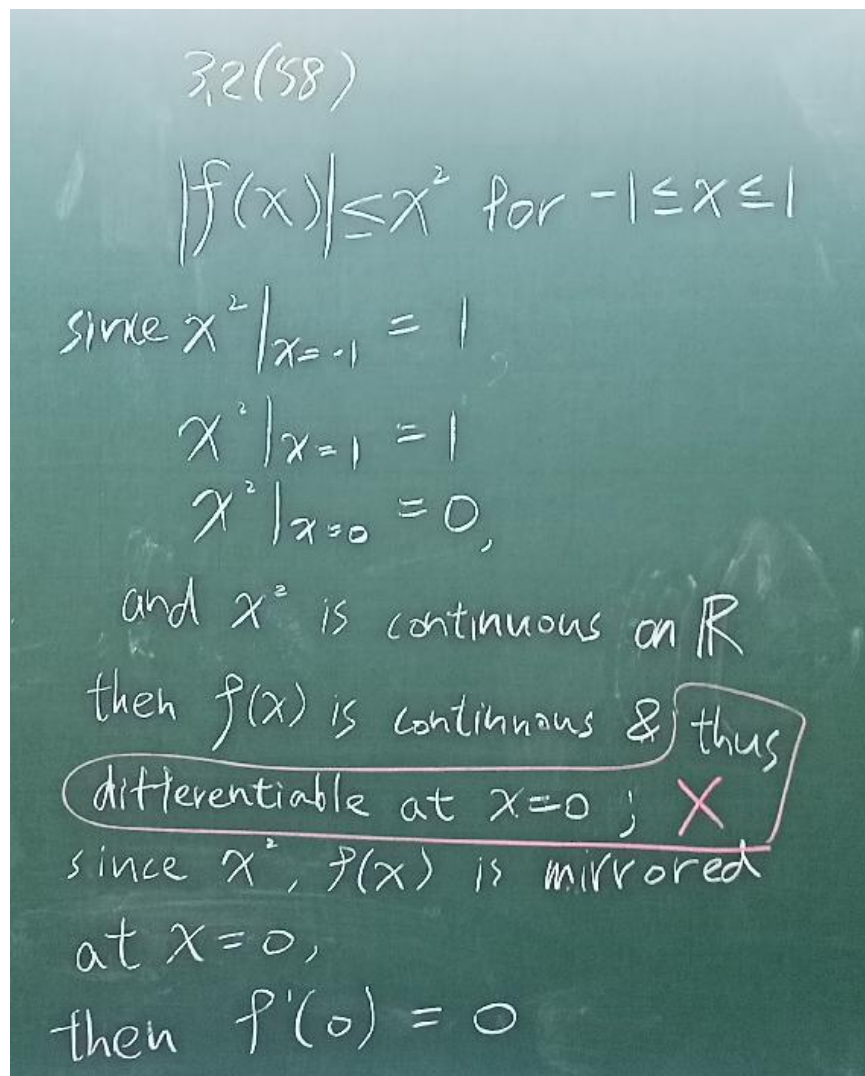


Figure 4: Common mistakes to Section 3.2, problem 58(a)

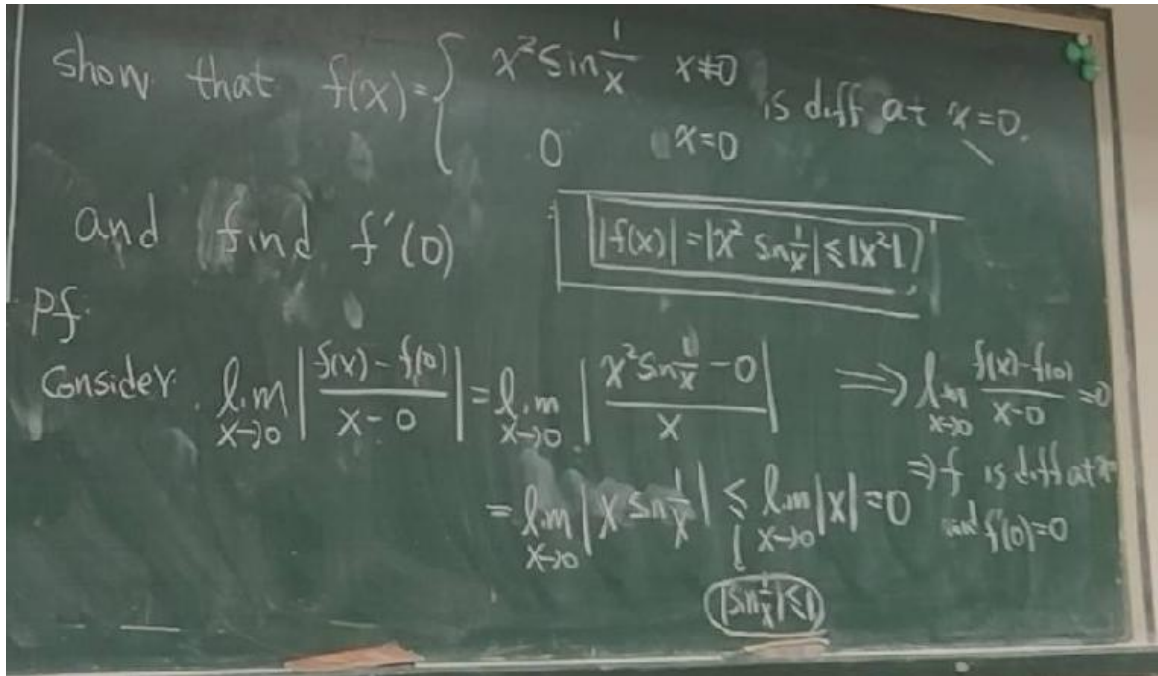


Figure 5: Solution to Section 3.2, problem 58(b)

2. Section 3.3: Solutions, common mistakes and corrections:

3.3 (23) 3.3 (

$$f(s) = \frac{\sqrt{s-1}}{\sqrt{s+1}}$$

$$f'(s) = \frac{(\sqrt{s+1})(\sqrt{s-1})' - (\sqrt{s+1})'(\sqrt{s-1})}{(\sqrt{s+1})^2}$$

$$= \frac{\frac{1}{2}\sqrt{s} - \frac{1}{2}s^{-\frac{1}{2}}}{(\sqrt{s+1})^2} = ? \Delta$$

$$f''(s) = \frac{(\sqrt{s+1})^2 \cdot A' - (\sqrt{s+1}) \cdot \frac{1}{\sqrt{s}} \cdot A}{(\sqrt{s+1})^4}$$

trick $f(s) = 1 - \frac{2}{\sqrt{s+1}}$

$$f'(s) = -\frac{0 - 2 \cdot \frac{1}{2} s^{-\frac{1}{2}}}{(\sqrt{s+1})^2} \quad \checkmark$$

Figure 6: A trick for Section 3.3, problem 23

$$\begin{aligned} & 3.3 (47) \\ & \left(\frac{(\theta-1)(\theta^2+\theta+1)}{\theta^3} \right) \\ & = \frac{\theta^3 - 1}{\theta^3} \\ & = 1 - \frac{1}{\theta^3} \\ & \therefore f'(\theta) = 3\theta^{-4} \end{aligned}$$

Figure 7: A trick for Section 3.3, problem 47

a. $y' = 3x^2 - 4$,
 $x = 2 \Rightarrow 3 \cdot 2^2 - 4 = 8 \neq$

b. $y'' = 6x$, ①
 when $x = 0$, $y'_{\min} = -4 \neq$
 ② $(0, 1)$

c. $y' = 3x^2 - 4 = 8$
 $x = -2, y = 1$
 (1) $y - 1 = 8(x + 2)$
 $y = 8x + 15 \neq$
 (2) $y - 1 = 8(x - 2)$
 $y = 8x - 15 \neq$

Normal line:
 $\frac{y-1}{x-2} = \frac{-1}{8}$

Figure 8: Solution to Section 3.3, problem 55

3.3 (67)

$$\lim_{x \rightarrow 1} \frac{(x-1)(x^{49} + x^{48} + \dots + 1)}{x-1}$$

$$= \lim_{x \rightarrow 1} x^{49} + x^{48} + \dots + 1 = 50 \quad \checkmark$$

$$\lim_{x \rightarrow 1} \frac{x^{50} - 1}{x - 1}$$

$$= \frac{d}{dx} x^{50} \Big|_{x=1}$$

$$= 50x^{49} \Big|_{x=1} = 50$$

Figure 9: A trick for Section 3.3, problem 67

3.3(70)

$f(x)$ is differentiable for all x -values, that is, $f(x)$ is continuous on \mathbb{R} .

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^+} f(x) = f(-1)$$

$$\Rightarrow -a + b = b - 3$$

$$\Rightarrow \underline{a = 3} \#$$

$$f'(x) = \begin{cases} a, & x > -1 \\ 2bx, & x \leq -1 \end{cases}$$

$$\Rightarrow a = -2b = 3 \Rightarrow \underline{b = -\frac{3}{2}} \#$$

Figure 10: Solution to Section 3.3, problem 70

3. Solution to Homework 04, problem 4,5:

4. $\frac{d}{dx} \begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix}$

$$= \frac{d}{dx} (f_{11}(x)f_{22}(x) - f_{12}(x)f_{21}(x))$$

$$= \underline{f_{11}'(x)f_{22}(x) + f_{11}(x)f_{22}'(x)} - \underline{f_{12}'(x)f_{21}(x) - f_{12}(x)f_{21}'(x)}$$

$$\rightarrow \begin{vmatrix} f_{11}'(x) & f_{12}'(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f_{21}'(x) & f_{22}'(x) \end{vmatrix}$$

$$\rightarrow \begin{vmatrix} f_{11}'(x) & f_{12}(x) \\ f_{21}'(x) & f_{22}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}'(x) \\ f_{21}(x) & f_{22}'(x) \end{vmatrix}$$

5. $\frac{d}{dx}(u(x)v(x)) = uv' + u'v$

$$\frac{d^2}{dx^2}(u(x)v(x)) = uv'' + u'v' + u'v' + u''v = uv'' + 2u'v' + u''v$$

$$\frac{d^3}{dx^3}(u(x)v(x)) = uv''' + 3u'v'' + 3u''v' + u'''v$$

Claim $\frac{d^n}{dx^n} = \sum_{k=0}^n \binom{n}{k} u^{(k)} v^{(n-k)}$

$n=2$ hold, Assume $n=m$ hold

For $n=m+1$, $\frac{d^{m+1}}{dx^{m+1}}(u(x)v(x)) = \frac{d}{dx} \left(\frac{d^m}{dx^m}(u(x)v(x)) \right)$

$$= \frac{d}{dx} \left(\sum_{k=0}^m \binom{m}{k} u^{(k)} v^{(m-k)} \right)$$

$$= \sum_{k=0}^m \binom{m}{k} u^{(k)} v^{(m-k+1)} + \sum_{k=0}^m \binom{m}{k} u^{(k+1)} v^{(m-k)} = uv^{(m+1)} + \sum_{k=1}^m \left[\binom{m}{k} + \binom{m}{k-1} \right] u^{(k)} v^{(m-k+1)} + u^{(m+1)} v$$

$$= \sum_{k=0}^{m+1} \binom{m+1}{k} u^{(k)} v^{(m+1-k)} \quad \text{hold by Induction, Claim hold} \quad \#$$

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Figure 11: Solution to Homework 04, problem 4,5