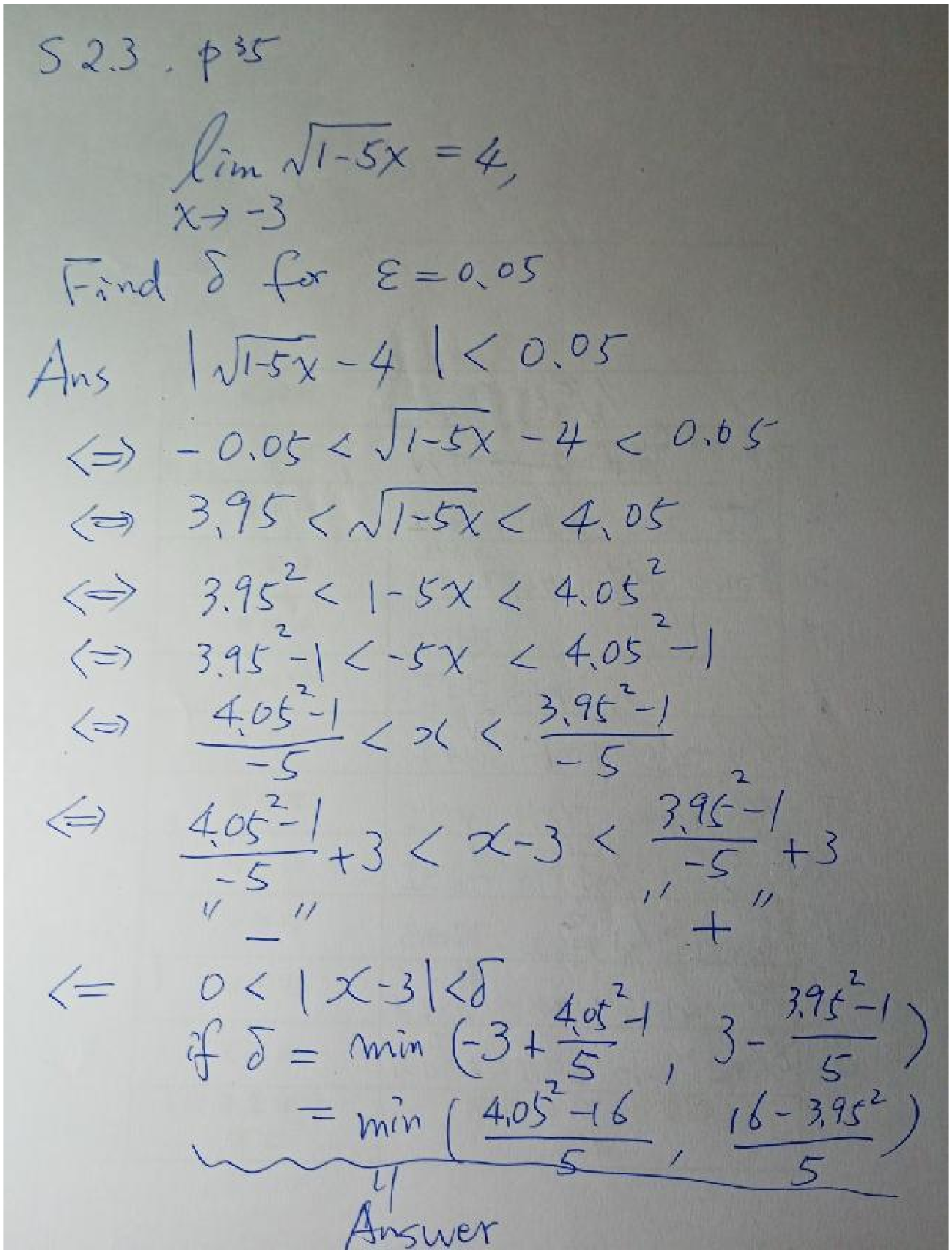
Brief solutions to selected problems in homework 02

1. Section 2.3: Solutions, common mistakes and corrections:



S 2.3 . p35

$$\lim_{x\to -3}\sqrt{1-5x}=4,$$

Find δ for $\varepsilon = 0.05$

Ans $\left|\sqrt{1-5x}-4\right| < 0.05$

$\Leftrightarrow -0.05 < \sqrt{1-5x}-4 < 0.05$

$\Leftrightarrow 3.95 < \sqrt{1-5x} < 4.05$

$\Leftrightarrow 3.95^2 < 1-5x < 4.05^2$

$\Leftrightarrow 3.95^2-1 < -5x < 4.05^2-1$

$\Leftrightarrow \dfrac{4.05^2-1}{-5} < x < \dfrac{3.95^2-1}{-5}$

$\Leftrightarrow \underbrace{\dfrac{4.05^2-1}{-5}+3}_{-} < x-3 < \underbrace{\dfrac{3.95^2-1}{-5}+3}_{+}$

$\Leftarrow 0 < |x-3| < \delta$

if $\delta = \min\left(-3+\dfrac{4.05^2-1}{5},\ 3-\dfrac{3.95^2-1}{5}\right)$

$\underbrace{= \min\left(\dfrac{4.05^2-16}{5},\ \dfrac{16-3.95^2}{5}\right)}_{\text{Answer}}$

Figure 1: Section 2.3, problem 35

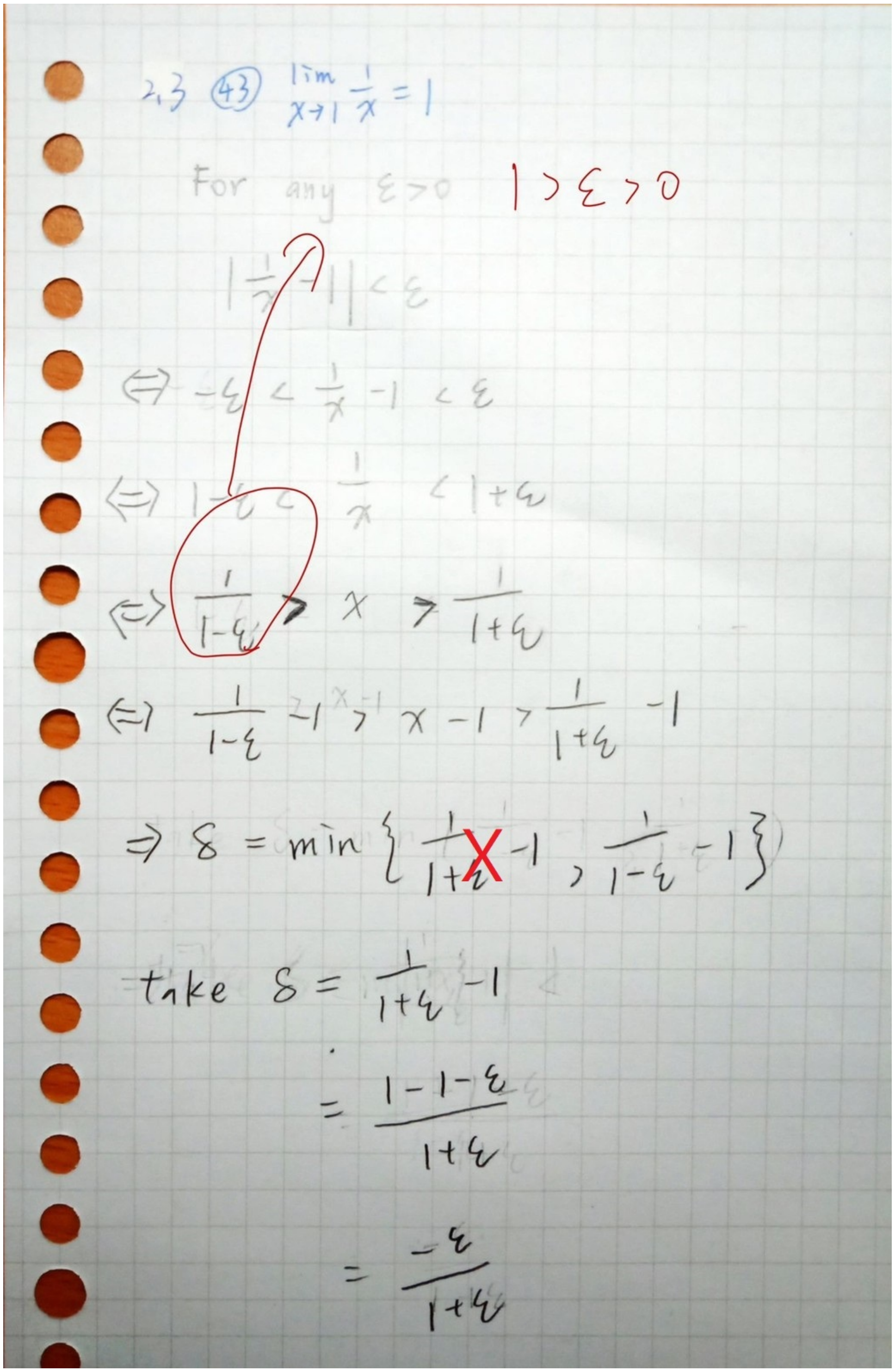


Figure 2: Section 2.3, problem 43: mistake 1

43.

$0<|x-1|<\delta$

$\Rightarrow -\delta<x-1<\delta$

$\Rightarrow -\delta+1<x<\delta+1$

$\left|\frac{1}{x}-1\right|<\varepsilon$ (A)

$\Leftrightarrow -\varepsilon<\frac{1}{x}-1<\varepsilon$

$\Leftrightarrow -\varepsilon+1<\frac{1}{x}<\varepsilon+1$ assume $0<\varepsilon<1$

$\Leftrightarrow \frac{1}{1+\varepsilon}<x<\frac{1}{1-\varepsilon}$ (B)

$$\begin{cases}\delta+1\le\frac{1}{1-\varepsilon}\\ -\delta+1\ge\frac{1}{1+\varepsilon}\end{cases}\Rightarrow\begin{cases}\delta\le\frac{1}{1-\varepsilon}-1\\ \delta\le 1-\frac{1}{1+\varepsilon}\end{cases}$$

take $\delta=\min\left\{\frac{1}{1-\varepsilon}-1,\ 1-\frac{1}{1+\varepsilon}\right\}$

因為要說明所選取的delta可以從(B)推到(A),反向箭頭是必要的

Figure 3: Section 2.3, problem 43: mistake 2

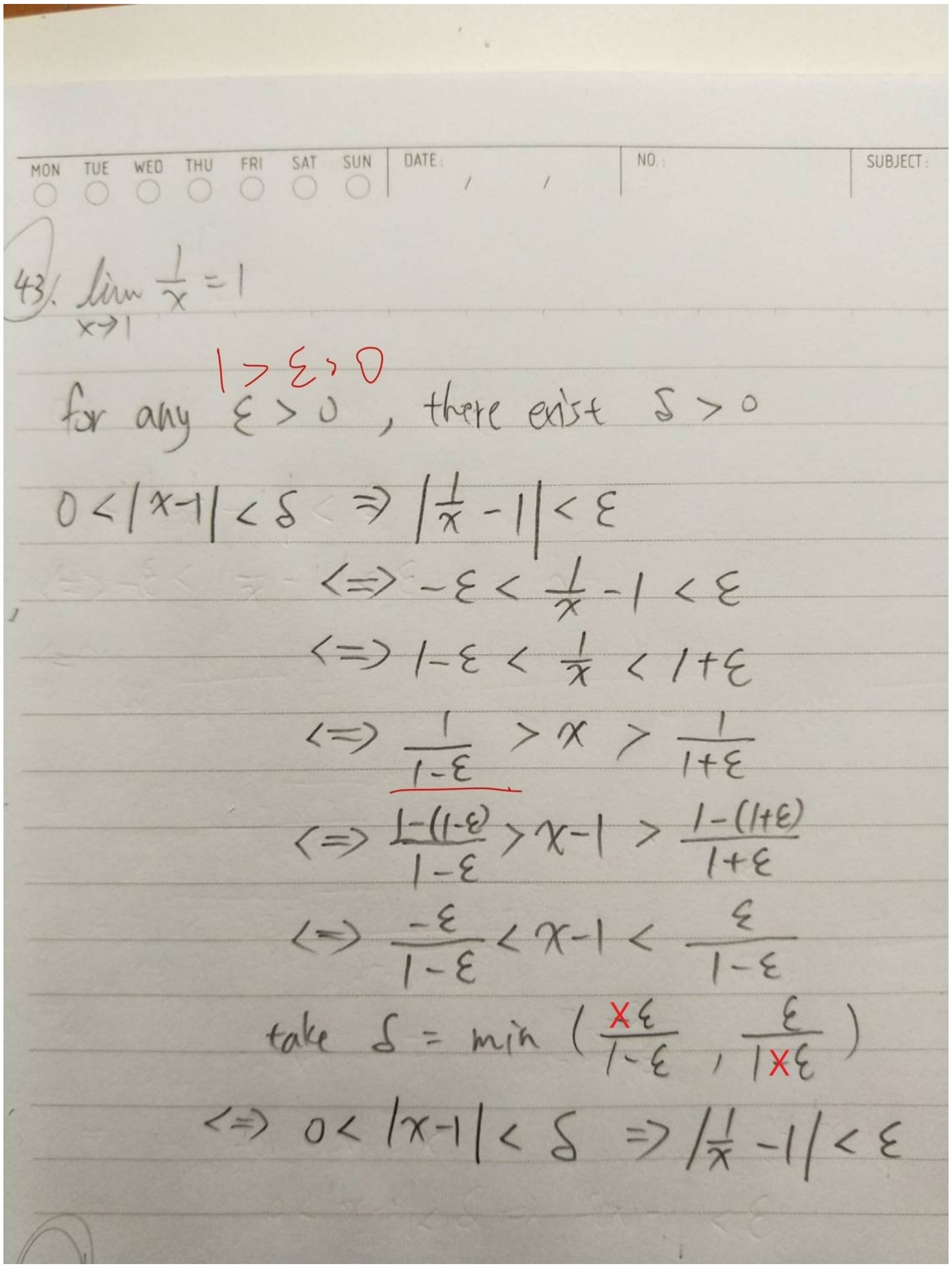


Figure 4: Section 2.3, problem 43: mistake 3

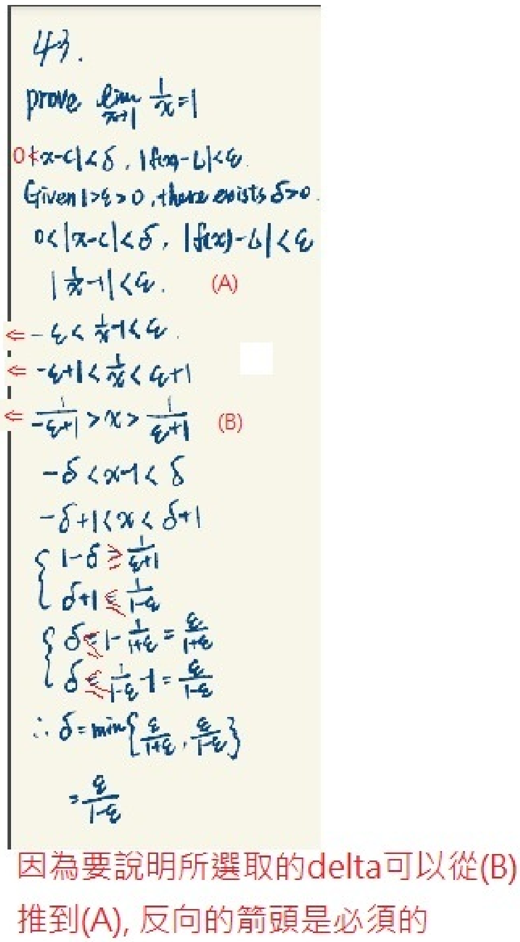


Figure 5: Section 2.3, problem 43: mistake 4

2. Problem 2: False. Counter example: $f(x) = \sin \dfrac{1}{x}$, $c = 0$, $L = 0$ satisfies the statment, but the limit does not exist.

3. Problem 3:

2. If $\lim\limits_{x \to c} f(x) = L$ and $\lim\limits_{x \to c} g(x) = M$

For any $\varepsilon > 0$, there exists $\delta_1 > 0$ and $\delta_2 > 0$, such that

$$0 < |x - c| < \delta_1 \Rightarrow |f(x) - L| < \frac{\varepsilon}{6}$$

$$0 < |x - c| < \delta_2 \Rightarrow |g(x) - M| < \frac{\varepsilon}{6}$$

$$\Leftrightarrow \begin{cases} -\frac{2}{3}\varepsilon < 4f(x) - 4L < \frac{2}{3}\varepsilon \\ -\frac{1}{3}\varepsilon < -2g(x) + 2M < \frac{1}{3}\varepsilon \end{cases}$$

take $\delta = \min(\delta_1, \delta_2)$

$$0 < |x - c| < \delta \Rightarrow -\varepsilon < 4f(x) - 2g(x) - (4L - 2M) < \varepsilon$$

$$\Leftrightarrow |4f(x) - 2g(x) - (4L - 2M)| < \varepsilon$$

$$\Rightarrow \lim_{x \to c} \left(4f(x) - 2g(x)\right) = 4L - 2M$$

Figure 6: Homework 02, problem 3