

Brief solutions to selected problems in homework 01

1. Section 2.2: Solutions, common mistakes and corrections:

sec 2.2, 81

If $\lim_{x \rightarrow 2} \frac{f(x)-5}{x-2} = 3$, then find $\lim_{x \rightarrow 2} f(x) = ?$

Sol: By observation we have $\lim_{x \rightarrow 2} f(x) = ?$

Now since both $\lim_{x \rightarrow 2} (x-2)$ and $\lim_{x \rightarrow 2} \frac{f(x)-5}{x-2}$ exist, by limit law

we have

$$\begin{aligned} \lim_{x \rightarrow 2} f(x) &= \lim_{x \rightarrow 2} (x-2) \lim_{x \rightarrow 2} \frac{f(x)-5}{x-2} + \lim_{x \rightarrow 2} 5 \\ &= \lim_{x \rightarrow 2} 5 = 5 \quad \square \end{aligned}$$
$$\begin{aligned} \lim_{x \rightarrow 2} f(x) &= \lim_{x \rightarrow 2} \left((x-2) \frac{f(x)-5}{x-2} + 5 \right) \\ &= \lim_{x \rightarrow 2} \left((x-2) \frac{f(x)-5}{x-2} + 5 \right) \end{aligned}$$

Figure 1: Section 2.2, problem 81

Remark: The idea: Since the (limit of the) ratio is nonzero and the (limit of the) denominator is zero, it follows that the (limit of the) numerator must be zero, otherwise the ratio will diverge. The idea can be supported by the Limit Laws as detailed above.