

$$\text{Eg 1} \int \frac{\cos \sqrt{\theta}}{\sqrt{\theta} \sin \sqrt{\theta}} d\theta$$

$$= \int \frac{\cos \sqrt{\theta}}{\sin \sqrt{\theta}} \frac{d\theta}{\sqrt{\theta}}$$
$$\theta^{\frac{-1}{2}} d\theta = 2 d\theta^{\frac{1}{2}}$$

$$= 2 \int \frac{\cos \sqrt{\theta}}{\sin \sqrt{\theta}} d\sqrt{\theta}$$

$$\cos \sqrt{\theta} d\sqrt{\theta} = d \sin \sqrt{\theta}$$

$$= 2 \int \frac{d \sin \sqrt{\theta}}{\sin \sqrt{\theta}}$$

$$= \ln |\sin \sqrt{\theta}| + C$$

Eg 2  $\int \sqrt{\frac{x-1}{x^5}} dx, x > 0$

$$\frac{dx}{\sqrt{x}} = 2d\sqrt{x} \quad (\times)$$

$$\frac{dx}{x} = d\ln x \quad (\times)$$

$$\frac{dx}{x^{\frac{3}{2}}} = -2d x^{\frac{1}{2}} \quad (\times)$$

$$\frac{dx}{x^2} = -dx^{-1} \quad (0)$$

$$= \int \sqrt{\frac{x-1}{x}} \frac{dx}{x^2}$$

$$= - \int \sqrt{1-x^{-1}} d x^{-1}$$

$$= \int \sqrt{1-x^{-1}} d(1-x^{-1}) = \frac{2}{3} (1-x^{-1})^{\frac{3}{2}} + C$$

$$\text{Ex 3 } \int (x+5)^2 (x-5)^{\frac{1}{3}} dx$$

$$\text{Let } x-5 = y, \quad x+5 = y+10$$

$$= \int (y+10)^2 y^{\frac{1}{3}} dy$$

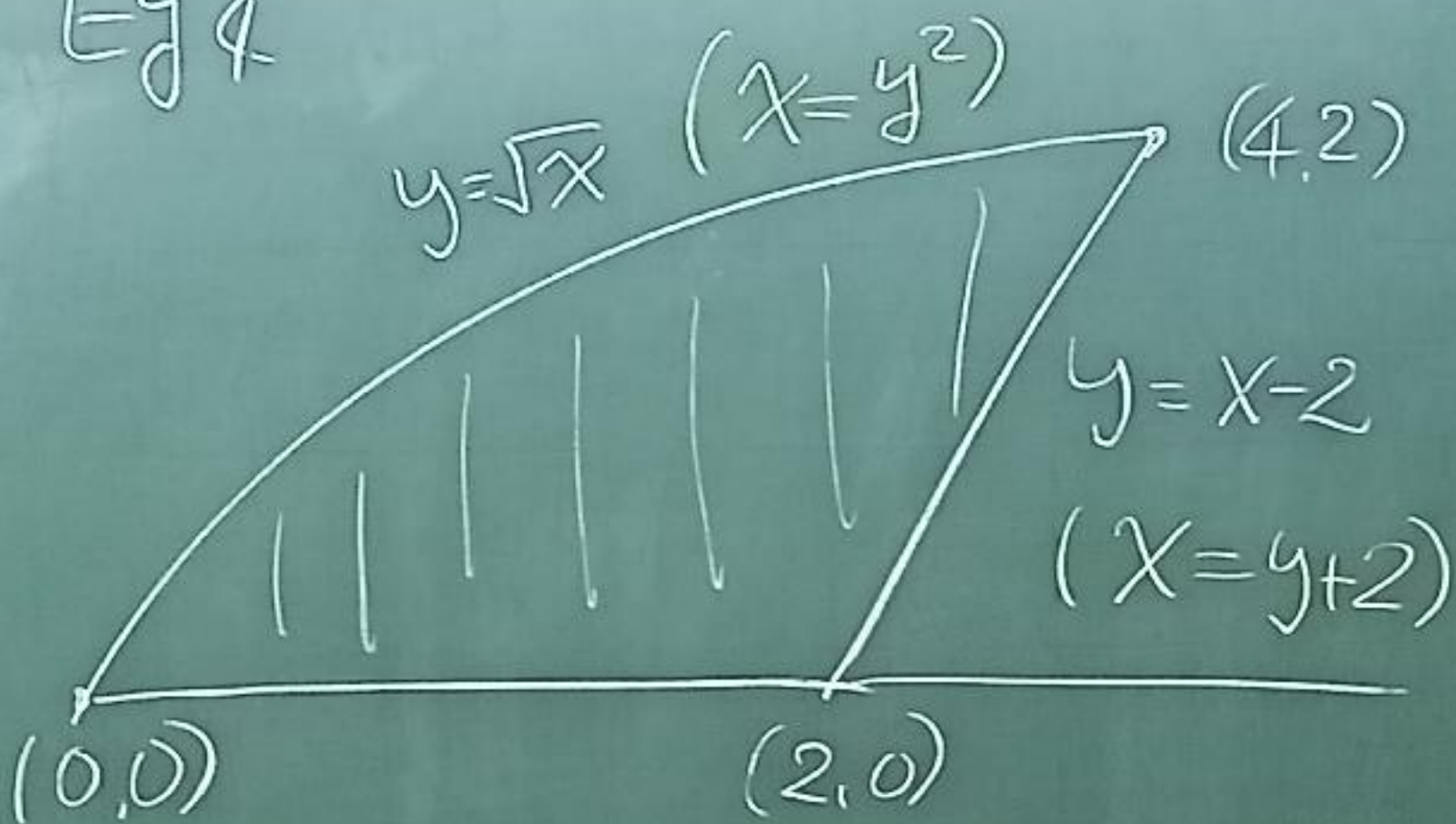
$$= \int y^{\frac{7}{3}} + 20y^{\frac{4}{3}} + 100y^{\frac{1}{3}} dy$$

$$= \frac{3}{10} y^{\frac{10}{3}} + \frac{60}{7} y^{\frac{7}{3}} + 75 y^{\frac{4}{3}} + C$$

$$= \frac{3}{10} (x-5)^{\frac{10}{3}} + \frac{60}{7} (x-5)^{\frac{7}{3}} + 75 (x-5)^{\frac{4}{3}} + C$$

# Area between Curves

Eg 4



$$A = \text{[shaded region 1]} + \text{[shaded region 2]} \quad \left( \int dx \right) \quad \text{(I)}$$

$$= \text{[shaded region 1]} - \text{[shaded region 2]} \quad \left( \int dx \right) \quad \text{(II)}$$

$$= \text{[shaded region 1]} \quad \left( \int dy \right) \quad \text{(III)}$$

