Recall

$$\frac{d}{dy}\ln y = \frac{1}{y}.\quad y>0$$

i.e. $\frac{d}{dx}\ln x = \frac{1}{x}.\quad x>0.$

Application: if $u(x)>0$

then $\frac{d}{dx}\ln u(x) = \frac{1}{u(x)}\cdot u'(x)$

Eg 1 $\frac{d}{dx}\ln(x^2+3) = \frac{2x}{x^2+3}$

Rm If $x<0$

$$\frac{d}{dx}\ln(-x) = \frac{-1}{-x} = \frac{1}{x}$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x}.\quad x\neq 0.$$

Eg2. $a>0$, $\frac{d}{dx}a^x = ?$

<u>Ans</u>: $a^x = (e^{\ln a})^x = e^{(\ln a)x}$

$$\therefore \frac{d}{dx}a^x = \frac{d}{dx}e^{(\ln a)x}$$
$$= e^{(\ln a)x} \cdot \frac{d}{dx}((\ln a)x)$$
$$= a^x \cdot \ln a \quad \cdots (*)$$

<u>Remark</u>: We showed that
$$\frac{d}{dx}a^x = g(a) \cdot a^x$$
where $g(a) = \lim_{h \to 0} \frac{a^h - 1}{h}$

now we know $g(a) \overset{(*)}{=} \ln a$

Eg3 $\frac{d}{dx} 3^{\sin x}$

$= \frac{d}{dx} e^{(\ln 3)\cdot \sin x}$

$= e^{(\ln 3)\sin x} \cdot \frac{d}{dx}((\ln 3)\sin x)$

$= 3^{\sin x} \cdot \ln 3 \cdot \cos x$

Eg4, $a>0$, $a \neq 1$; $x>0$

$\frac{d}{dx} \log_a x = \frac{d}{dx} \frac{\log_e x}{\log_e a}$

$= \frac{d}{dx} \frac{\ln x}{\ln a} = \frac{1}{x \ln a}$

Similarly, $a>0$, $u(x)>0$

$\frac{d}{dx} \log_a u(x) = \frac{u'(x)}{u(x) \ln a}$

Eg 4. $y = \dfrac{(x^2+1)(x+3)^{\frac{1}{2}}}{x-1}, x>1.\ y' = ?$

$$\ln y = \ln(x^2+1) + \frac{1}{2}\ln(x+3) - \ln(x-1)$$

$\frac{d}{dx}$: $$\frac{y'}{y} = \frac{2x}{x^2+1} + \frac{1}{2}\frac{1}{(x+3)} - \frac{1}{x-1}$$

$$y' = y \cdot \left(\frac{2x}{x^2+1} + \frac{1}{2}\frac{1}{(x+3)} - \frac{1}{x-1} \right)$$

$$= \frac{(x^2+1)(x+3)^{\frac{1}{2}}}{(x-1)} \cdot \left(\frac{2x}{x^2+1} + \frac{1}{2(x+3)} - \frac{1}{x-1} \right)$$

Eg5. $x>0,\ n\in\mathbb{R}$

$$\frac{d}{dx}x^n=\frac{d}{dx}\left(e^{\ln x}\right)^n$$

$$=\frac{d}{dx}e^{n\ln x}=e^{n\ln x}\frac{d}{dx}(n\ln x)$$

$$=x^n\cdot\frac{n}{x}=nx^{n-1}$$

Eg6: $x>0,\ \frac{d}{dx}x^x=?$

<u>Ans</u> $=\frac{d}{dx}\left(e^{\ln x}\right)^x=\frac{d}{dx}e^{x\ln x}$

$$=e^{x\ln x}\cdot\frac{d}{dx}(x\ln x)$$

$$=x^x\cdot(\ln x+1)$$

Eg7: $e = \lim_{x \to 0} (1+x)^{\frac{1}{x}}$

$= \lim_{m \to \infty} \left(1+\frac{1}{m}\right)^m$

pf: $\lim_{x \to 0} (1+x)^{\frac{1}{x}}$

$= \lim_{x \to 0} e^{\frac{\ln(1+x)}{x}}$

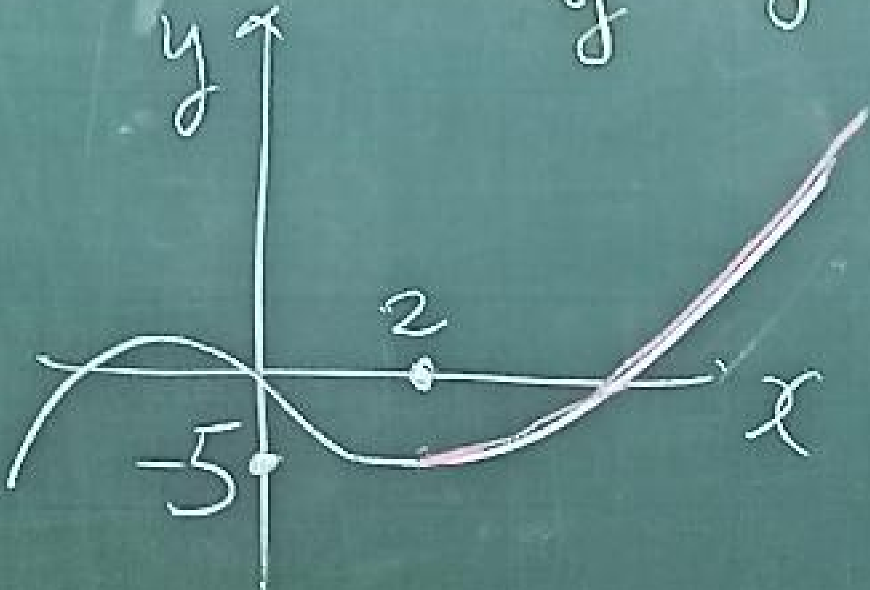
$= e^{\left(\lim_{x \to 0} \frac{\ln(1+x)}{x}\right)}$

$= e^{\lim_{x \to 0}\left(\frac{\ln(1+x) - \ln 1}{x - 0}\right)}$

$= e^{\frac{d}{dx}\ln(1+x)\big|_{x=0}} = e^{\frac{1}{1+0}} = e$

Eg8 $f(x)=x^3-3x^2-1$, $\underline{x\ge 2}$

Find $\frac{d}{d\underset{y}{\cancel{x}}}f^{-1}(\underset{y}{\cancel{x}})$ at $\underset{y}{\cancel{x}}=-1=f(3)$



$\underline{\text{Ans}}$ $\frac{d}{dy}f^{-1}(y)\Big|_{y=f(3)}$

$$=\frac{1}{f'(x)\big|_{x=3}} \quad \left(\ne \frac{1}{f'(-1)}\right)$$

$$=\frac{1}{3x^2-6x}\Big|_{x=3}=\frac{1}{9}$$