Homework 07

1. Section 3.7:

   Let f^{-1} be the inverse function of f. Suppose f and f^{-1} are twice differentiable (i.e. both first and second derivative exist). Derive the formula of $\left(f^{-1}\right)''$ in terms of f', f'' and f^{-1}.

2. Section 3.9: problems 9, 11, 21, 23, 25, 33, 35, 39, 53, 55.

3. Section 3.11: problems 9, 11, 16(c,d), 17, 53, 55, 65(a,b,f(for $f(x)$ only)), 66.

4. A key point in section 3.11 is that the error of linear approximation, $f(x) - L(x)$, satisfies

   $$\lim_{x \to a} \frac{f(x) - L(x)}{x - a} = 0 \tag{1}$$

   provided f is differentiable at $x = a$.

   The following statement gives more details about the error $f(x) - L(x)$ and will be introduced in the near future. Take this statement for granted for now:

   > If f is twice differentiable near $x = a$, then
   >
   > $$f(x) - L(x) = \frac{1}{2} f''(c)(x - a)^2 \tag{2}$$
   >
   > for some c between x and a.

   From (2), we have an error bound

   $$|f(x) - L(x)| \le \frac{1}{2} \left(\max_{c \text{ between } x \text{ and } a} |f''(c)| \right) (x - a)^2 \tag{3}$$

   Use (3) to estimate the error of linear approximation (i.e. find out $|f(x) - L(x)| \le \cdots$) for problem 17 (b) of Section 3.11.

5. Chapter 3, Practice Exercises (p230-p231): problems 96, 125, 129.

6. Chapter 3, additional and advanced problems (p235): problems 16, 21, 22(d), 23.