Homework Assignment for Chapter 03

1. Section 3.2: problems 17, 48, 54, 57.
2. Read about second and higher order derivatives at end of section 3.3.
3. Section 3.3: problems 47, 55, 67, 70, 75(c).
4. Use product rule to show (and memorize) that

$$\frac{d}{dx}\begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix} = \begin{vmatrix} f'_{11}(x) & f_{12}(x) \\ f'_{21}(x) & f_{22}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f'_{12}(x) \\ f_{21}(x) & f'_{22}(x) \end{vmatrix}$$
$$= \begin{vmatrix} f'_{11}(x) & f'_{12}(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f'_{21}(x) & f'_{22}(x) \end{vmatrix}$$

   What are the corresponding formulae for 3 by 3 determinants, 4 by 4 determinants, etc.?
5. Apply the product rule repeatedly to get $\frac{d^n}{dx^n}\Big(u(x)v(x)\Big)$ in terms of derivatives of $u(x)$ and $v(x)$. Start with $n = 2$, then $n = 3, \cdots$, to find the formula.
6. Section 3.5: problems 17, 33(a), 34(a), 49 (Hint: what is $\frac{d\sin\theta}{d\theta}\Big|_{\theta=c}$?) , 57, 58.
7. Section 3.6: Do as many as time permits from problems 51, 53, $\cdots$, 77.
8. Assume $g(2) = 3$, $g'(2) = 0.1$, $f'(2) = 3$, $f'(3) = 4$ and $f'(4) = 5$. What is $\frac{d}{dx}f(g(x))$ at $x = 2$?
9. Section 3.7: problems 27, 31, 48, 51(a).
10. Section 3.8: problems 8, 9, 37, 39, 51 (Hint: take ln on both sides first), 65, 77, 89, 91, 93, 95, 98.
11. Section 3.9: problems 9, 11, 21, 23, 25, 33, 35, 39, 53, 55.
12. Section 3.11: problems 9, 11, 16(c,d), 17, 53, 55, 65(a,b,f(for $f(x)$ only)), 66.
13. A key point in section 3.11 is that the error of linear approximation, $f(x) - L(x)$, satisfies

$$\lim_{x\to a}\frac{f(x) - L(x)}{x - a} = 0 \tag{1}$$

    provided f is differentiable at $x = a$.

    The following statement gives more details about the error $f(x) - L(x)$ and will be introduced in the near future. Take this statement for granted for now:

If f is twice differentiable near $x = a$, then

$$f(x) - L(x) = \frac{1}{2} f''(c)(x-a)^2 \tag{2}$$

for some c between x and a.

From (2), we have an error bound

$$|f(x) - L(x)| \leq \frac{1}{2} \left(\max_{c \text{ between } x \text{ and } a} |f''(c)| \right) (x-a)^2 \tag{3}$$

Use (3) to estimate the error of linear approximation (i.e. find out $|f(x) - L(x)| \leq$???) for problem 17 (b) of Section 3.11.

14. Chapter 3, additional and advanced problems: problems 16, 21, 22(d), 23.