Homework Assignment for Chapter 02

1. Section 2.2: 33, 41, 63, 77, 81, 87.

   Remark: For problem 87, just find the limit. Need not plot it. To find the limit, use the algebraic identity '$a^3 - b^3 = \cdots$' mentioned in lecture 01 video.

2. Section 2.3: problems 35, 43, 49, 53.

3. Use the $\varepsilon - \delta$ argument to prove the following statement:

   If $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$, then $\lim_{x\to c} \Big(4f(x) - 2g(x)\Big) = 4L - 2M$.

   Hint: Note that if $a < b$ then $-a > -b$.

4. Suppose that $f(x)$ is defined on $(c-a, c) \cup (c, c+a)$ for some $a > 0$. If $f(x)$ satisfies the following statement, then is it true that $\lim_{x\to c} f(x) = L$? Prove it if true, find a counter example if not true.

   > For any $\varepsilon > 0$ and any $\delta > 0$, there exists a number $x \in (c-\delta, c) \cup (c, c+\delta)$ such that $|f(x) - L| < \varepsilon$.

5. (Optional) Section 2.3: Problems 57 (see derivations of this definition in Lecture 03).

6. Section 2.4: Problems 26, 34, 42, 48.

7. Chap 2, Additional and Advanced Exercises: Problems 25 (Hint: $1 - \cos x = 2\sin^2 \frac{x}{2}$), 26 on page 136.

8. Section 2.5: problems 37 (just find the limit, ignore the continuity question) 64, 67, 68, 77 (Need not graph it).

9. Section 2.6: problems 61, 85, 92, 93, 95, 97.

10. Study definition of the limits in p102, p119, p125 and p131. How would you define the four limits

    $$\lim_{x\to\pm\infty} f(x) = \pm\infty?$$

    Verify the statement

    $$\lim_{x\to\infty} -x^3 = -\infty$$

    using the definition you wrote.