Study guide for Chap 16

1. Section 16.1, 16.2:

   All exam problems will be expressed in explicit mathematical symbols. So you do not need to memorize the definitions of first moments, center of mass and moments of inertia, etc. in Section 16.1 and work, circulation, flow and flux in Section 16.2 in preparing the exams.

2. Section 16.1, 16.2:

   Study the meanings of

   $$\int_C f(x,y,z)\,ds,$$

   $$\int_C \boldsymbol{F}(x,y,z)\cdot \boldsymbol{T}ds = \int_C \boldsymbol{F}(x,y,z)\cdot d\boldsymbol{r} = \int_C M(x,y,z)\,dx + N(x,y,z)\,dy + P(x,y,z)\,dz,$$

   $$\int_C \boldsymbol{F}(x,y)\cdot \boldsymbol{n}ds = \oint_C M(x,y)\,dy - N(x,y)\,dx$$

   and how to calculate them using a properly chosen parametrization of C: $\boldsymbol{r}(t)$, $t_0 \le t \le t_1$.

   A few points to pay attention:

   Which of them is (are) independent of the orientation of C? Which of them depend(s) on the orientation of C?

   How do you choose the parametrization $\boldsymbol{r}(t)$ so that the direction of $\boldsymbol{T}$ comply with the orientation of C?

   How is the outward normal $\boldsymbol{n}$ related to $\boldsymbol{T}$ if the parametrization of C is increasing in the counter-clockwise direction?

3. Section 16.3:

   Study and memorize the definitions of 'path independent', 'conservative' and 'potential function' (p984).

   Study the proof of Theorem 1:'Fundamental Theorem of Line Integrals' (p985).

   Study the definition and examples (in pictures) of 'connected domain' and 'simply connected domain' (p985).

   Study the proof of Theorem 2: 'Conservative Fields are Gradient Fields' (p986).

   Regarding the two theorems on equivalent characterization of conservative fields, Theorem 3: 'Loop Property' (p987) and 'Component test' (p988), which of them requires the domain to be simply connected? If the domain is not simply connected, which of the implications '$\Longleftarrow$', '$\Longrightarrow$' remains valid? which one may probably be false? Is Example 5 a counter example of which case?

   For given functions $M(x,y,z), N(x,y,z), P(x,y,z)$ satisfying the component test, how does one find the potential function (if it exists) by way of direct integration (Example 3)?

4. Section 16.4:

   Study and memorize definitions of divergence and curl. It is easier to start with

$$\text{div}\boldsymbol{F}(x,y,z) = \nabla \cdot \boldsymbol{F}(x,y,z), \quad \text{and} \quad \text{curl}\boldsymbol{F}(x,y,z) = \nabla \times \boldsymbol{F}(x,y,z),$$

   then reduce them to

$$\text{div}\boldsymbol{F}(x,y) \quad \text{and} \quad \text{curl}\boldsymbol{F}(x,y)\cdot \boldsymbol{k}.$$

   Study and memorize Green's Theorem both in tangential form and normal form (p1000).

   Is Theorem 4 applicable to Example 5 of Section 16.3? Which part went wrong?

5. Section 16.5, 16.6:

   Study how to calculate $\iint_S G(x,y,z)\,d\sigma$ when the surface S is given by the parametrization

$$\boldsymbol{r}(u,v) = f(u,v)\,\boldsymbol{i} + g(u,v)\,\boldsymbol{j} + h(u,v)\,\boldsymbol{k}, \quad a \le u \le b, \quad c \le v \le d.$$

   Study how to calculate $\iint_S G(x,y,z)\,d\sigma$ when the surface S is given by a graph

$$x = f(y,z), \text{ (or implicitly by } F(x,y,z)=0), \quad (y,z) \in \mathcal{R},$$

   or a graph

$$y = g(x,z), \text{ (or implicitly by } F(x,y,z)=0), \quad (x,z) \in \mathcal{R},$$

   or a graph

$$z = h(x,y), \text{ (or implicitly by } F(x,y,z)=0), \quad (x,y) \in \mathcal{R}.$$

6. Section 16.7:

   Memorize the definition of curl operator $\nabla\times$. Memorize the statement of Stokes' Theorem. Find a few examples and practice to verify the statement (that is compute both the surface integral and line integral and check they are the same).

   Study the meaning and proof of the identity $\nabla \times \nabla f = \boldsymbol{0}$.

   Pay attention to examples where $\nabla \times \boldsymbol{F} = \boldsymbol{0}$ in D, but $\oint_C \boldsymbol{F}\cdot\boldsymbol{T}\,ds \neq 0$ for some closed loop $C \subset D$.

7. Section 16.8:

   Memorize the definition of divergence operator $\nabla\cdot$. Memorize the statement of the Divergence Theorem. Find a few examples and practice to verify the statement (that is compute both the surface integral and triple integral and check they are the same).

   Study the meaning and proof of the identity $\nabla \cdot \nabla \times \boldsymbol{F} = 0$.

   Pay attention to examples where $\nabla\cdot\boldsymbol{F} = 0$ in D, but $\iint_S \boldsymbol{F}\cdot\boldsymbol{n}\,d\sigma \neq 0$ for some closed surface $S \subset D$.