Study guide for Chap 15

1. Section 15.1, 15.2: Study how to identify the limits of integration in $\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dxdy$ and $\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dydx$ for general domains (that is, not rectangles).

2. Section 15.2, 15.3: Study how to interchange between $\int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dxdy$ and $\int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dydx$ for general domains as in problems 33-56 of section 15.2.

3. Section 15.4:

   Study why $dA = rdrd\theta$ in polar coordinates. Practice how to determine the limits of integration in $\int_{\theta_1}^{\theta_2} \int_{f_1(\theta)}^{f_2(\theta)} (\cdots) rdrd\theta$ as in Examples 2-6 of section 15.4. More specifically:

   (a) Given a domain R in the $x-y$ plane, practice drawing $\theta = C$ lines in R. The end points of these lines are lower limit (the near end point) and upper limit (the far end point) of the integration rdr. The end points for $d\theta$ are smallest and largest C among these $\theta = C$ lines.

   (b) The end points of the lines $\theta = C$ must be expressed as $r = f_1(\theta)$ and $r = f_2(\theta)$. Given a simple curve $F(x,y) = 0$ (such as a line or a circle), use the substitution $x = r\cos\theta$, $y = r\cos\theta$ to express it as $r = f(\theta)$. Examples: $x = 1$, $y = -3$, $x + y = 1$, $x^2 + y^2 = 4$, etc.

4. Section 15.5:

   Practice how to determine the limits of integration for triple integrals in rectangular coordinates. For example, which cross section ($\{x = \text{constant}\}$, $\{y = \text{constant}\}$ or $\{x = \text{constant}\}$) is needed for $dxdydz$? which cross section is needed for $dzdxdy$? etc.

   On a cross section, the triple integral reduces to double integral for the first two integration variables.

   Note that, the upper and lower limits of the first variable may depend on the second and third variables. The upper and lower limits of the second variable may depend on the third variable.

   Practice this on corresponding examples and exercises in section 15.5.

5. Section 15.7:

   Practice on drawing cross section $\{r = \text{constant}\}$, $\{\theta = \text{constant}\}$ and $\{z = \text{constant}\}$ in cylindrical coordinates. Which one is needed for $drd\theta dz$? which one is needed for $dzdrd\theta$? etc.

   On a cross section, the triple integral reduces to double integral for the first two integration variables. For example, on a $\{z = \text{constant}\}$ cross section, it reduces to double integral $drd\theta$. One can then follow item 4 above to determine the upper and lower limits of integration.

   Repeat the same practice in spherical coordinates.

   Also note that, the upper and lower limits of the first variable may depend on the second and third variables. The upper and lower limits of the second variable may depend on the third variable.

6. Section 15.8:

   Study the meaning of the Jacobian and memorize the formula both in double and triple integration.

7. Section 15.8:

   Assume that a change of variables between (x, y, z) and (u, v, w) is given, study how to change the lower and upper limits of integration for (x, y, z) into lower and upper limits of integration for (u, v, w). See the examples in section 15.8.