$F(x,y) = (x, 0)$, $\frac{x^2}{4} + y^2 = 1$, $\begin{cases} x = 2\cos t \\ y = \sin t \end{cases}$, $0 \le t \le 2\pi$ (10)

$$\int_C F \cdot n \, ds = \int_C M\,dy - N\,dx$$

$$= \int_0^{2\pi} 2\cos t \cdot \cos t \, dt - 0 \quad (10)$$

$$= \int_0^{2\pi} 1 + \cos 2t \, dt = 2\pi. \quad (13)$$

2. <I> $\nabla\left(2\sqrt{x^2+2y^2+3z^2}\right) = F$

$\Rightarrow$ F is conservative $\Rightarrow$ F satisfys component test.

<II> $\dfrac{\partial}{\partial y}\left(\dfrac{2x}{\sqrt{x^2+2y^2+3z^2}}\right) = \dfrac{-4xy}{\sqrt{x^2+2y^2+3z^2}^{\,3}} = \dfrac{\partial}{\partial x}\left(\dfrac{4y}{\sqrt{x^2+2y^2+3z^2}}\right)$

Similarly, for $\begin{cases} \dfrac{\partial}{\partial z}\left(\dfrac{2x}{\sqrt{x^2+2y^2+3z^2}}\right) = \dfrac{\partial}{\partial x}\left(\dfrac{6z}{\sqrt{x^2+2y^2+3z^2}}\right) \\ \dfrac{\partial}{\partial z}\left(\dfrac{4y}{\sqrt{x^2+2y^2+3z^2}}\right) = \dfrac{\partial}{\partial y}\left(\dfrac{6z}{\sqrt{x^2+2y^2+3z^2}}\right) \end{cases}$ 28 pts

And $R^3 \setminus \{(0,0,0)\}$ is simply connected. — 5 pts

3. <I> $\displaystyle\int_C F\cdot T\,ds = 2\sqrt{x^2+2y^2+3z^2}\Big|_{(1,1,1)}^{(3,4,5)} = 2\sqrt{116} - 2\sqrt{6}$

<II> $r_1(t): (1,1,1) \to (2,2,2)$
$r_2(t): (2,2,2) \to (3,4,5)$ } 10 pts

$\displaystyle\int_{r_1} F\cdot T\,ds = 2\sqrt{6}$ —— 10 pts

$\displaystyle\int_{r_2} F\cdot T\,ds = 2\sqrt{116} - 4\sqrt{6}$ —— 10 pts

$\displaystyle\int_C F\cdot T\,ds = 2\sqrt{116} + 2\sqrt{6}$ — 4 pts