Quiz 6

Apr 26, 2022:

Name: _____ Student ID number: _____ Classroom: _____

1. (a) Find the tangent plane to the surface $x^2 + 2y^2 + 3z^2 = 6$ at the point $(1, 1, 1)$.

   (b) Find a tangent vector to the curve given by the intersection of the two surfaces $xyz = 1$ and $x^2 + 2y^2 + 3z^2 = 6$ at the point $(1, 1, 1)$.

2. Find the linearization of the $f(x, y) = x^2 - xy + y^2 - 6x + 2$ near $(1, 2)$ and give an upper bound for the error $|E|$ of this approximation on the rectangle $0 \le x \le 2, 1 \le y \le 3$.

3. Find all critical points of $f(x, y) = x^2 - xy + y^2 - 6x + 2$ and determine whether they are local min, local max or saddle points.

1.(a) $f_x = 2x$. $f_y = 4y$. $f_z = 6z$. $\nabla f(1,1,1) = (2, 4, 6)$ 10pts

→ Tangent plane : $x + 2y + 3z = 6$. 10pts.

(b) $(\nabla xyz)(1,1,1) = (1, 1, 1)$. $-(1, 1, 1) \times (1, 2, 3) = (-1, 2, -1)$.
10 pts 10 pts

2. $\nabla f = (2x - y - 6, -x + 2y)$. $\nabla f(1,2) = (-6, 3)$.

⇒ linearization : $L(x,y) = f(1,2) - 6(x-1) + 3(y-2) = -1 - 6(x-1) + 3(y-2)$ 15

$f_{xx} = 2$. $f_{xy} = -1$. $f_{yy} = 2$. ⇒ bounds $|f_{xx}|$. $|f_{xy}|$. $|f_{yy}|$ } 15pts.

⇒ $|E| \le \frac{1}{2} \cdot 2 \cdot (|x-1| + |y-2|)^2 = 4$
5pts

3. $\nabla f = (2x - y - 6, -x + 2y)$. $\nabla f = 0 \Rightarrow (x, y) = (4, 2)$ 10 pts.

$f_{xx} f_{yy} - f_{xy}^2 = 3 > 0$. $f_{xx} > 0$ ⇒ $f(4,2)$ local min.
15pts 5pts.