Brief solutions to Quiz 4

Apr 12, 2022.

1. (40 pts) Give formal definition of $\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$. Does $f(x,y) = \dfrac{x^2y}{x^4+y^2}$ have a limit at $(0,0)$? Explain.

   Ans:

   Definition: See page 816 of the textbook.

   No. See Section 14.2, Example 6 on page 820 of the textbook.

2. (40 pts)

   (a) Give formal definition of '$f(x,y)$ is differentiable at (x_0,y_0)'.

   (b) True or False? Explain:
   If $f(x,y)$ is differentiable at (x_0,y_0), then it is continuous at (x_0,y_0).

   Ans:

   (a): See page 832 of the textbook, or page 2 of Lecture 13 note. Either one will do.

   (b): True. See page 832 of the textbook, or page 9 of Lecture 13 note.

3. (20 pts) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at $(x,y,z) = (1,2,3)$ if $z(x,y)$ is implicitly defined by $x^2 + \dfrac{y^2}{4} + \dfrac{z^2}{9} = 3$.

   Ans:

$$\frac{\partial}{\partial x} \implies 2x + \frac{2z}{9} z_x = 0 \implies z_x = \frac{-9x}{z} = -3$$
$$\frac{\partial}{\partial y} \implies \frac{y}{2} + \frac{2z}{9} z_y = 0 \implies z_y = \frac{-9y}{4z} = -\frac{3}{2}$$