

Brief solutions to Quiz 3

Mar 15, 2022:

1. (12+12+12 pts. Correct answer: 4 pts. Correct explanation: 8 pts)

Are the following series convergent? Explain.

(a)
$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

Ans: Convergent. Since it converges absolutely (by direct comparison with the p -series, $p = 2$).

(b)
$$\sum_{n=1}^{\infty} \frac{n!n!}{(2n)!}$$

Ans: Convergent by ratio test ($\rho = \frac{1}{4}$).

(c)
$$\sum_{n=2}^{\infty} \frac{n}{(\ln n)^n}$$

Ans: Convergent by root test ($\rho = 0$).

2. (Correct statement: 8 pts. Correct example: 8 pts. Correct explanation: 16 pts)

State the Alternating Series Test (need not prove it) and give an example of a conditionally convergent series (give details).

Ans:

Statement: See Theorem 15, Section 10.6.

Example: $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$. It is convergent by the Alternating Series Test. It does not converge absolutely (the p -series with $p = 1$).

3. (Correct definition: 8 pts. Correct example: 8 pts. Correct explanation: 16 pts)

Suppose $0 < R < \infty$. Give the definition of

(The radius of convergence for $\sum_{n=1}^{\infty} a_n(x-1)^n = R$)

For the definition part, explain what it means in terms of convergence/divergence of the power series (but not how to find R). Then give an example of a power series whose radius of convergence is "2" (give details).**Ans:**Definition: If the above series converges (absolutely) on $|x-1| < R$ and diverges $|x-1| > R$.Example: $\sum_{n=1}^{\infty} \left(\frac{x}{2}\right)^n$ (details skipped).