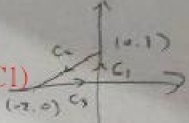
tan: $\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$



(Mdx+Ndy= 0 on C1)

LHS $= \underbrace{\int_0^2 -M(-t, 1-\tfrac{t}{2})\,dt}_{C_2} + \underbrace{\int_0^2 M(t-2, 0)\,dt}_{C_3}$

(N=0) (N=0)

$u=-t$, $v=t-2$: $= -\int_{-2}^{0} M(u, 1+\tfrac{u}{2})\,du + \int_{-2}^{0} M(v, 0)\,dv$

RHS $= \int_{-2}^{0}\int_{0}^{1+\frac{x}{2}} -M_y(x,y)\,dy\,dx$

$= -\int_{-2}^{0} M(x, 1+\tfrac{x}{2})\,dx + \int_{-2}^{0} M(x,0)\,dx$

$\therefore \oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$

C_1: $x=0$, $y=t$, $0 \le t \le 1$

C_2: $x=-t$, $y=1-\frac{t}{2}$, $0 \le t \le 2$

C_3: $x=t-2$, $y=0$, $0 \le t \le 2$

norm: $\oint_C M\,dy - N\,dx = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\,dy$

C_1, C_3 same

C_2: $x = 2(1-t)-2$, $y = 1-t$, $0 \le t \le 1$

LHS $= \oint_{C_1} M\,dy + \oint_{C_2} M\,dy$ (Mdy-Ndx=0 on C3)

(N=0) (N=0)

$u=t$, $v=1-t$: $= \int_0^1 M(0,t)\,dt - \int_0^1 M(-2t, 1-t)\,dt = \int_0^1 M(0,u)\,du - \int_0^1 M(2v-2, v)$

RHS $= \int_0^1 \int_{2y-2}^{0} M_x(x,y)\,dx\,dy$

$= \int_0^1 M(0,y)\,dy - \int_0^1 M(2y-2, y)\,dy$