19. $F = (y, -x, x^2)$, $r(t) = (2\cos t, 2\sin t, 3 - 2\cos^2 t)$.

$0 \le t \le 2\pi$

$$\oint F \cdot dr = \int_0^{2\pi} \begin{pmatrix} y \\ -x \\ x^2 \end{pmatrix} \begin{pmatrix} -2\sin t \\ 2\cos t \\ +6\cos^2 t \sin t \end{pmatrix} dt.$$

$$= \int_0^{2\pi} -4 + 24\cos^4 t \sin t \, dt = -8\pi$$

$$\iint_S \nabla \times F \cdot n \, d\sigma = \iint_{\text{◎}} \nabla \times F \cdot n \, d\sigma + \iint_{\text{▨}} \nabla \times F \cdot n \, d\sigma$$

$$\nabla \times F = (0, -2x, -2).$$

$$\iint_{\text{◎}} \nabla \times F \cdot n \, d\sigma = \iint_{\text{◎}} \begin{pmatrix} 0 \\ -2x \\ -2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} d\sigma = \iint_{x^2+y^2 \le 4} -2 \, d\sigma = -8\pi.$$

$$\iint_{\text{▨}} \nabla \times F \cdot n \, d\sigma = \iint_{\text{▨}} \begin{pmatrix} 0 \\ -2x \\ -2 \end{pmatrix} \begin{pmatrix} -x \\ -y \\ 0 \end{pmatrix} \frac{d\sigma}{\sqrt{x^2+y^2}}$$

$r(\theta, z) = (2\cos\theta, 2\sin\theta, z)$

$0 \le \theta \le 2\pi$, $0 \le z \le 2 - \cos^3\theta$.

$\Rightarrow |r_\theta \times r_z| = 2$

$$\Rightarrow \iint_{\text{▨}} \frac{2xy}{\sqrt{x^2+y^2}} \, d\sigma = \int_0^{2\pi} \int_0^{2-\cos^3\theta} \frac{2(2\cos\theta)(2\sin\theta)}{2} \, 2 \, dz \, d\theta.$$

$$= 0.$$

Thus $\oint F \, dr = \iint_S \nabla \times F \cdot n \, d\sigma$