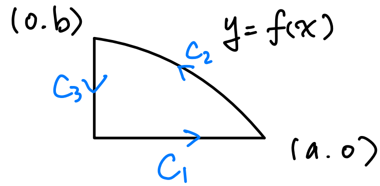
4. $F = (M(x,y),\ \underbrace{0}_{N=0})$ on R



$C_1: \begin{cases} x = t \\ y = 0 \end{cases} \quad 0 \le t \le a$

$C_2: \begin{cases} x = a-t \\ y = f(a-t) \end{cases} \quad 0 \le t \le a$

$C_3: \begin{cases} \boxed{x=0} \\ y = b-t \end{cases} \quad 0 \le t \le b$

Tangential form :

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy$$

$$= \int_0^a \int_0^{f(x)} -\frac{\partial M}{\partial y}\, dy\,dx$$

$$= \int_0^a (-M(x, f(x)) + M(x, 0))\, dx$$

$$= -\int_0^a M(x, f(x))\, dx + \int_0^a M(x, 0)\, dx$$

$$\oint_C M\,dx - N\,dy$$

$$= \oint_{C_1} M\,dx + \oint_{C_2} M\,dx + \cancelto{0}{\oint_{C_3} M\,dx}$$

$$= \int_0^a M(t, 0)\, dt + \int_0^a M(a-t, f(a-t))(-dt)$$

let $x = a-t$

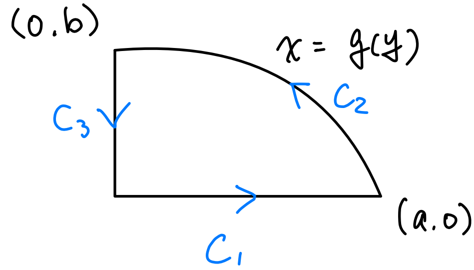
$dx = -dt$

$$= \int_0^a M(t, 0)\,dt + \int_a^0 M(x, f(x))\, dx$$

$$= \int_0^a M(x, 0)\, dx - \int_0^a M(x, f(x))\, dx$$

$$\therefore \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy = \oint_C M\,dx + N\,dy$$

Normal form:



$$\iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\,dy$$

$$= \int_0^b \int_0^{g(y)} \frac{\partial M}{\partial x}\,dx\,dy$$

$$= \int_0^b \big(M(g(y), y) - M(0, y)\big)\,dy$$

$$= \int_0^b M(g(y), y)\,dy - \int_0^b M(0.y)\,dy$$

$$C_1 : \begin{cases} x = t \\ \boxed{y = 0} \end{cases} \quad 0 \le t \le a$$

$$C_2 : \begin{cases} x = g(t) \\ y = t \end{cases} \quad 0 \le t \le b$$

$$C_3 : \begin{cases} x = 0 \\ y = b - t \end{cases} \quad 0 \le t \le b$$

$$\oint_C M\,dy - N\,dx$$

$$= \oint_{C_1} \cancelto{0}{M\,dy} + \oint_{C_2} M\,dy + \oint_{C_3} M\,dy$$

$$= \int_0^b M(g(t).t)\,dt + \int_0^b M(0.b-t)(-dt)$$

let $y = b - t$
$dy = -dt$

$$= \int_0^b M(g(t).t)\,dt + \int_b^0 M(0.y)\,dy$$

$$= \int_0^b M(g(y).y)\,dt - \int_0^b M(0.y)\,dy$$

$$\therefore \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\,dy = \oint_C M\,dy - N\,dx$$