I. (10.3)

$$F = \frac{x}{\sqrt{x^2+y^2}} i + \frac{y}{\sqrt{x^2+y^2}} j + 0k$$

$$G = \frac{-y}{x^2+y^2} i + \frac{x}{x^2+y^2} j + 0k$$

a $M_1 = \frac{x}{\sqrt{x^2+y^2}}$, $N_1 = \frac{y}{\sqrt{x^2+y^2}}$, $P_1 = 0$

$$\frac{\partial P_1}{\partial y} = 0 = \frac{\partial N_1}{\partial z}, \quad \frac{\partial P_1}{\partial x} = 0 = \frac{\partial M_1}{\partial z}, \quad \frac{\partial M_1}{\partial y} = -\frac{xy}{(x^2+y^2)^{\frac{3}{2}}} = \frac{\partial N_1}{\partial x}$$

$\Rightarrow$ F satisfies the component test

$M_2 = \frac{-y}{x^2+y^2}$, $N_2 = \frac{x}{x^2+y^2}$, $P_2 = 0$

$$\frac{\partial P_2}{\partial y} = 0 = \frac{\partial N_2}{\partial z}, \quad \frac{\partial P_2}{\partial x} = 0 = \frac{\partial M_2}{\partial z}, \quad \frac{\partial M_2}{\partial y} = \frac{y^2-x^2}{(x^2+y^2)^2} = \frac{\partial N_2}{\partial x}$$

$\Rightarrow$ G satisfies the component test b $\nabla f = F$

$$\frac{\partial f}{\partial x} = M_1 \,, \frac{\partial f}{\partial y} = N_1 \,, \frac{\partial f}{\partial z} = P_1$$

$$f(x,y,z) = \sqrt{x^2+y^2} + g(y,z)$$

$$\frac{y}{\sqrt{x^2+y^2}} + \frac{\partial g}{\partial y} = \frac{y}{\sqrt{x^2+y^2}} \Rightarrow \frac{\partial g}{\partial y} = 0$$

$$\Rightarrow f(x,y,z) = \sqrt{x^2+y^2} + h(z)$$

$$0 + \frac{\partial h}{\partial z} = 0 \Rightarrow \frac{\partial h}{\partial z} = 0 \,, \quad h(z) = \cancel{z + C} \; C$$

$$\Rightarrow f(x,y,z) = \sqrt{x^2+y^2} \cancel{+z} + C$$

C $r(t) = c(\cos t)\,i + (\sin t)\,j \;,\; 0 \le t \le 2\pi$

$$G = \frac{-y}{x^2+y^2}\, i + \frac{x}{x^2+y^2}\, j$$

$$= \frac{-\sin t}{\sin^2 t + \cos^2 t}\, i + \frac{\cos t}{\sin^2 t + \cos^2 t}\, j$$

$$= (-\sin t)\, i + (\cos t)\, j$$

$$\frac{dr}{dt} = (-\sin t)\, i + (\cos t)\, j$$

$$\oint G \cdot dr = \oint_c G \cdot \frac{dr}{dt}\, dt$$

$$= \int_0^{2\pi} (\sin^2 t + \cos^2 t)\, dt$$

$$= 2\pi \neq 0$$

$\because \oint G \cdot dr \neq 0$

$\therefore$ G isn't conservative by Thm 3.

(d) Find if there exists h, a differential function, such that $H = \nabla h$