

1b-4.

#10.

$$\vec{F} = (\underbrace{x+3y}_M, \underbrace{2x-y}_N)$$

$$C: x^2 + y^2 = 2$$

$$\sqrt{2} \cos t$$

$$r(t) = (\cancel{2 \cos t}, \sqrt{2} \sin t) \quad 2\pi \leq t < 2\pi$$

$$\text{use Green's Thm} = \oint \vec{F}(r(t)) \cdot r'(t) dt = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} dx dy (\text{flow})$$

$$\oint M dy - N dx = \iint_R \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} dx dy (\text{flux})$$

$$\text{circulation} = \iint_R (2-3) dx dy$$

$$= \int_{-1}^1 \int_{-\sqrt{2-2y^2}}^{\sqrt{2-2y^2}} (-1) dx dy$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{\frac{2-x^2}{2}}}^{\sqrt{\frac{2-x^2}{2}}} (-1) dy dx$$

$$= -\frac{2}{\sqrt{2}} \int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{2-x^2} dx$$

$\Rightarrow$ 圓上 $\frac{\pi}{4}$ ($r=\sqrt{2}$)

$$= -\sqrt{2} \pi$$

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$$\text{flux} = \iint_R \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = \iint_R 0 dx dy = 0$$

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