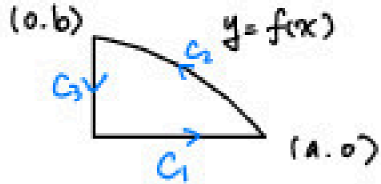
4. $F = (M(x,y),\ \underbrace{0}_{N=0})$ on R



$C_1 : \begin{cases} x = t \\ y = 0 \end{cases} \quad 0 \le t \le a$

$C_2 : \begin{cases} x = a-t \\ y = f(a-t) \end{cases} \quad 0 \le t \le a$

$C_3 : \begin{cases} \boxed{x=0} \\ y = b-t \end{cases} \quad 0 \le t \le b$

Tangential form :

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy$$

$$= \int_0^a \int_0^{f(x)} -\frac{\partial M}{\partial y}\, dy\,dx$$

$$= \int_0^a \left(-M(x, f(x)) + M(x,0)\right) dx$$

$$= -\int_0^a M(x, f(x))\,dx + \int_0^a M(x,0)\,dx$$

$$\oint_C M\,dx - N\,dy$$

$$= \oint_{C_1} M\,dx + \oint_{C_2} M\,dx + \cancel{\oint_{C_3} M\,dx} \to 0$$

$$= \int_0^a M(t,0)\,dt + \int_0^a M(a-t, f(a-t))(-dt)$$

let $x = a-t$

$dx = -dt$

$$= \int_0^a M(t,0)\,dt + \int_a^0 M(x, f(x))\,dx$$

$$= \int_0^a M(x,0)\,dx - \int_0^a M(x, f(x))\,dx$$

$$\therefore \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy = \oint_C M\,dx + N\,dy$$