2. Let $\vec{F} = \frac{1}{\sqrt{x^2+y^2+z^2}}(x, y, z)$.

(a) What is the natural domain D_F of $\vec{F}$?

(b) Show that $\vec{F}$ satisfies component test in D_F.

(c) Is D_F simply connected?

(d) Is $\vec{F}$ conservative in this domain?

(a)
$$D_F = \{(x, y, z) \mid x^2+y^2+z^2 > 0\} = \mathbb{R}^3 \setminus \{(0,0,0)\}$$

(b)
$$\frac{\partial}{\partial y}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) = -xy(x^2+y^2+z^2)^{-\frac{3}{2}} = \frac{\partial}{\partial x}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right)$$
$$\frac{\partial}{\partial z}\left(\frac{x}{\sqrt{x^2+y^2+z^2}}\right) = -xz(x^2+y^2+z^2)^{-\frac{3}{2}} = \frac{\partial}{\partial x}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$
$$\frac{\partial}{\partial z}\left(\frac{y}{\sqrt{x^2+y^2+z^2}}\right) = -yz(x^2+y^2+z^2)^{-\frac{3}{2}} = \frac{\partial}{\partial y}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)$$

$\therefore \vec{F}$ satisfies component test in D_F

(c)
D_F is simply connected

(d)
By (b), $\vec{F}$ satisfies component test in D_F.

Also, D_F is simply connected.

$\therefore \vec{F}$ is conservative in D_F