Sec 16.3

21. $\int_{(1,1,1)}^{(2,2,2)} \frac{1}{y}\,dx + \left(\frac{1}{z} - \frac{x}{y^2}\right)dy + \left(-\frac{y}{z^2}\right)dz.$

Find $f$ satisfys $\nabla f = \left(\frac{1}{y}, \frac{1}{z} - \frac{x}{y^2}, -\frac{y}{z^2}\right)$

$\Rightarrow f_x = \frac{1}{y} \Rightarrow f = \frac{x}{y} + g(y,z)$ —① for some $g: R^2 \to R$

$f_y = \frac{1}{z} - \frac{x}{y^2}$, by ① $f_y = -\frac{x}{y^2} + g_y \Rightarrow g_y = \frac{1}{z} \Rightarrow g = \frac{y}{z} + h(z)$ —②

$f_z = -\frac{y}{z^2}$, by ①,② $f_z = 0 + \left(-\frac{y}{z^2}\right) + h'(z) \Rightarrow h'(z) = 0 \Rightarrow h(z) = C$.

for some constant $C$.

$\Rightarrow f = \frac{x}{y} + \frac{y}{z} + C$. take $f = \frac{x}{y} + \frac{y}{z}$.

$$\int_{(1,1,1)}^{(2,2,2)} \frac{1}{y}\,dx + \left(\frac{1}{z} - \frac{x}{y^2}\right)dy + \left(-\frac{y}{z^2}\right)dz = \left.\frac{x}{y} + \frac{y}{z}\right|_{(1,1,1)}^{(2,2,2)} = 0.$$