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31. Let D be the region in Exercise 11. Set up the triple integrals in spherical coordinates that give the volume of D using the following orders of integration.

a. $d\rho\, d\phi\, d\theta$ **b.** $d\phi\, d\rho\, d\theta$

11. Let D be the region bounded below by the plane $z = 0$, above by the sphere $x^2 + y^2 + z^2 = 4$, and on the sides by the cylinder $x^2 + y^2 = 1$. Set up the triple integrals in cylindrical coordinates that give the volume of D using the following orders of integration.

a. $dz\, dr\, d\theta$ **b.** $dr\, dz\, d\theta$ **c.** $d\theta\, dz\, dr$

a. $x^2+y^2=1$

$\Rightarrow \rho^2\sin^2\phi = 1$

$\rho\sin\phi = 1$

$\rho = \csc\phi$

$$\Rightarrow \int_0^{2\pi}\int_0^{\frac{\pi}{6}}\int_0^{2} \rho^2\sin\phi\, d\rho\, d\phi\, d\theta + \int_0^{2\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\int_0^{\csc\phi} \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

b. $$\int_0^{2\pi}\int_1^{2}\int_{\cancel{0}\ \frac{\pi}{6}}^{\sin^{-1}(\frac{1}{\rho})} \rho^2\sin\phi\, d\phi\, d\rho\, d\theta$$

$$+\int_0^{2\pi}\int_0^{2}\int_0^{\frac{\pi}{6}} \rho^2\sin\phi\, d\phi\, d\rho\, d\theta$$

$$+\int_0^{2\pi}\int_0^{1}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \rho^2\sin\phi\, d\phi\, d\rho\, d\theta$$