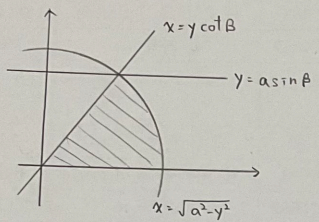
4. Chap 15-12

a.



$x = r\cos\theta \quad y = r\sin\theta$

$$\int_0^{a\sin\beta}\int_{y\cot\beta}^{\sqrt{a^2-y^2}} \ln(x^2+y^2)\,dx\,dy$$

$$= \int_0^{\beta}\int_0^{a} \ln r^2 \, r\,dr\,d\theta$$

$$\because\ r^2 = t \qquad 2r\,dr = dt$$

$$= \int_0^{\beta}\int_0^{a^2} \frac{1}{2}\ln t\,dt\,d\theta$$

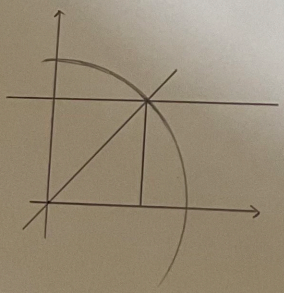
$$\because\ u = \ln t \quad du = \frac{1}{t}dt,\ dv = dt,\ v = t$$

$$= \frac{1}{2}\int_0^{\beta}\left(\left[t\ln t\right]_0^{a^2} - \int_0^{a^2} t\cdot\frac{1}{t}\,dt\right)d\theta$$

$$= \frac{1}{2}\int_0^{\beta}(2a^2\ln a - a^2)\,d\theta$$

$$= \frac{1}{2}(2a^2\ln a - a^2)\beta = a^2\beta\left(\ln a - \frac{1}{2}\right)$$

b.



$$0 \le x \le a\cos\beta$$

$$0 \le y \le x\cot\beta$$

$$a\cos\beta \le x \le a$$

$$0 \le y \le \sqrt{a^2-x^2}$$

$$\therefore \int_0^{a\sin\beta}\int_{y\cot\beta}^{\sqrt{a^2-y^2}} \ln(x^2+y^2)\,dx\,dy = \int_0^{a\cos\beta}\int_0^{x\cot\beta} \ln(x^2+y^2)\,dy\,dx + \int_{a\cos\beta}^{a}\int_0^{\sqrt{a^2-x^2}} \ln(x^2+y^2)\,dy\,dx$$