

10.3 53

Let $\{a_n\}$ be an nonincreasing sequence

$(a_n \geq a_{n+1})$ for all n of positive terms

that $\rightarrow 0$

$$\sum_{n=1}^{\infty} a_n = a_1 + \underline{a_2 + a_3} + \underline{a_4 + a_5} + \underline{a_6 + a_7} + \dots$$

$$\leq a_1 + \underline{a_2 + a_3} + \underline{a_4 + a_5} + \underline{a_6 + a_7} + \dots$$

$$= a_1 + \underline{2a_2} + \underline{2a_4} + \dots$$

$$= \sum_{n=0}^{\infty} 2^n a_{2^n} \quad \text{so by } \sum_{n=0}^{\infty} 2^n a_{2^n} \text{ converges}$$