Brief solutions to selected problems in homework week 14

1. Section 16.3, problem 21:

Sec 16.3

21. $\displaystyle\int_{(1,1,1)}^{(2,2,2)} \frac{1}{y}\,dx + \left(\frac{1}{z} - \frac{x}{y^2}\right)dy + \left(-\frac{y}{z^2}\right)dz.$

Find f satisfys $\nabla f = \left(\frac{1}{y}, \frac{1}{z} - \frac{x}{y^2}, -\frac{y}{z^2}\right)$

$f_x = \frac{1}{y} \Rightarrow f = \frac{x}{y} + g(y,z)$ —① for some $g: \mathbb{R}^2 \to \mathbb{R}$

$f_y = \frac{1}{z} - \frac{x}{y^2}$, by ① $f_y = -\frac{x}{y^2} + g_y \Rightarrow g_y = \frac{1}{z} \Rightarrow g = \frac{y}{z} + h(z)$ —②

$f_z = -\frac{y}{z^2}$, by ①② $f_z = 0 + \left(-\frac{y}{z^2}\right) + h'(z) \Rightarrow h'(z) = 0 \Rightarrow h(z) = C$

for some constant C.

$\Rightarrow f = \frac{x}{y} + \frac{y}{z} + C$. take $f = \frac{x}{y} + \frac{y}{z}$.

$$\int_{(1,1,1)}^{(2,2,2)} \frac{1}{y}\,dx + \left(\frac{1}{z} - \frac{x}{y^2}\right)dy + \left(-\frac{y}{z^2}\right)dz = \frac{x}{y} + \frac{y}{z}\Big|_{(1,1,1)}^{(2,2,2)} = 0.$$

2. Section 16.3, problem 26:

Method 1: Since $\boldsymbol{F}$ satisfies the component test (need to check it!) and the domain $D = R^3 \setminus \{(0,0,0)\}$ is simply connected, from the 'Component Test for Conservative Fields' property on page 988, we know that $\boldsymbol{F}$ is conservative.

Method 2: Since f is explicitly found (no matter how one finds it) to satisty $\boldsymbol{F} = \nabla f$, we know that $\boldsymbol{F}$ is conservative:

26. find f s.t. $\nabla f = \left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right)$!

take $f = \sqrt{x^2+y^2+z^2}$.