

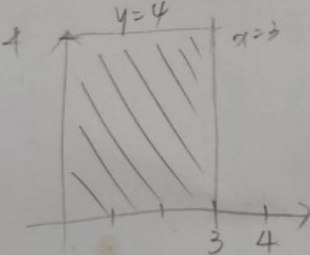
Brief solutions to selected problems in homework week 12

1. Section 15.4, problem 26:

15.4.26

$$\int_0^{\tan^{-1} \frac{4}{3}} \int_0^{3 \sec \theta} r^n dr d\theta + \int_{\tan^{-1} \frac{4}{3}}^{\pi/2} \int_0^{4 \csc \theta} r^n dr d\theta$$

$r \sin \theta = 4$
 $r \cos \theta = 3$
 $0 \leq \theta \leq \frac{\pi}{2}$

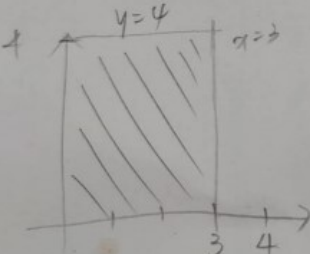
$$= \int_0^4 \int_0^3 (x^2 + y^2)^3 dx dy$$


2. Section 15.4, problem 42:

15.4.26

$$\int_0^{\tan^{-1} \frac{4}{3}} \int_0^{3 \sec \theta} r^n dr d\theta + \int_{\tan^{-1} \frac{4}{3}}^{\pi/2} \int_0^{4 \csc \theta} r^n dr d\theta$$

$r \sin \theta = 4$
 $r \cos \theta = 3$
 $0 \leq \theta \leq \frac{\pi}{2}$

$$= \int_0^4 \int_0^3 (x^2 + y^2)^3 dx dy$$


3. Section 15.5, problem 35:

$$\begin{aligned}
 V &= 2 \cdot \int_{-2}^2 \int_0^{\frac{1}{2}\sqrt{4-x^2}} \int_0^{x+2} dz dy dx \\
 &= 2 \int_{-2}^2 \int_0^{\frac{1}{2}\sqrt{4-x^2}} (x+2) dy dx = 2 \int_{-2}^2 (x+2) \frac{1}{2}\sqrt{4-x^2} dx \\
 &\quad \text{dy not dx} \downarrow \cancel{2} \int_{-2}^2 \left(\frac{1}{2}x^2 + 2x \right) \Big|_0^{\frac{1}{2}\sqrt{4-x^2}} dx \quad \int_{-2}^2 x\sqrt{4-x^2} dx \\
 &= 2 \int_{-2}^2 \frac{1}{8}(4-x^2) + \sqrt{4-x^2} dx \quad + \quad \int_{-2}^2 \sqrt{4-x^2} dx \\
 &= 4\pi
 \end{aligned}$$

4. Section 15.5, problem 41:

41. $\int_0^\pi \int_0^\pi \int_0^{\sqrt{xz}} \frac{\cos(xz)}{\sqrt{xz}} dz dy dx$

$= \int_0^\pi \int_0^\pi \int_0^{\sqrt{xz}} \frac{\cos(xz)}{\sqrt{xz}} dz dx dy = \int_0^\pi \int_0^\pi \frac{\cos(xz)}{\sqrt{xz}} dx dz$

$= \int_0^\pi \left(\frac{\sin(xz)}{z} \right) \Big|_0^{\sqrt{xz}} dz = \left[\frac{\sin(xz)}{z} \right]_0^\pi = \frac{\sin(\pi)}{\pi} = 0$

