

Brief solutions to selected problems in homework week 11

1. Section 14.10, problem 12:

12. $f(x, y, z, w) = 0 \dots \textcircled{1}$ $f = w^2x + w^2z + \frac{w^4}{y}$
 $g(x, y, z, w) = 0 \dots \textcircled{2}$
 $z = z(x, y), w = w(x, y)$
 By $\textcircled{1}$: $0 = \frac{\partial f}{\partial x} \frac{\partial x}{\partial x} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial x} + \frac{\partial f}{\partial w} \frac{\partial w}{\partial x}$ $\textcircled{3}$
 $\textcircled{2}$: $0 = \frac{\partial g}{\partial x} \frac{\partial x}{\partial x} + \frac{\partial g}{\partial z} \frac{\partial z}{\partial x} + \frac{\partial g}{\partial w} \frac{\partial w}{\partial x}$ $\textcircled{4}$

By $\textcircled{4}$: $\frac{\partial w}{\partial x} = \frac{1}{\frac{\partial g}{\partial w}} \left(-\frac{\partial g}{\partial x} - \frac{\partial g}{\partial z} \frac{\partial z}{\partial x} \right)$
 $\textcircled{3} \Rightarrow \frac{\partial f}{\partial x} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial x} + \frac{\partial f}{\partial w} \left(\frac{1}{\frac{\partial g}{\partial w}} \left(-\frac{\partial g}{\partial x} - \frac{\partial g}{\partial z} \frac{\partial z}{\partial x} \right) \right) = 0$
 $\left(\frac{\partial f}{\partial z} \right) \frac{\partial z}{\partial x} = \frac{\frac{\partial f}{\partial x} \frac{\partial g}{\partial w} - \frac{\partial f}{\partial w} \frac{\partial g}{\partial x}}{\frac{\partial g}{\partial z} \frac{\partial g}{\partial w} - \frac{\partial f}{\partial z} \frac{\partial g}{\partial w}}$

Remark:

$$f_x + f_w w_x + f_z z_x = 0$$

$$g_x + g_w w_x + g_z z_x = 0$$

Two linear equations for the two unknowns z_x and w_x , solve for z_x and w_x .

2. Problem 2:

5 variables (u, x, y, z, w) and 3 (equations+constraints). Therefore 2 independent variables (must be x, y since we want $(\frac{\partial u}{\partial x})_y$).

So $f(x, y, z(x, y), w(x, y)) = 0$, $g(x, y, z(x, y), w(x, y)) = 0$ and $u = U(x, y, z(x, y), w(x, y))$.

From the u equation, $u_x = U_x + U_z z_x + U_w w_x$.

From f and g constraints, we have as in problem 12:

$$f_x + f_w w_x + f_z z_x = 0$$

$$g_x + g_w w_x + g_z z_x = 0$$

then solve for w_x and z_x , plug them into $u_x = U_x + U_z z_x + U_w w_x$ to get the desired formula of $(\frac{\partial u}{\partial x})_y$.