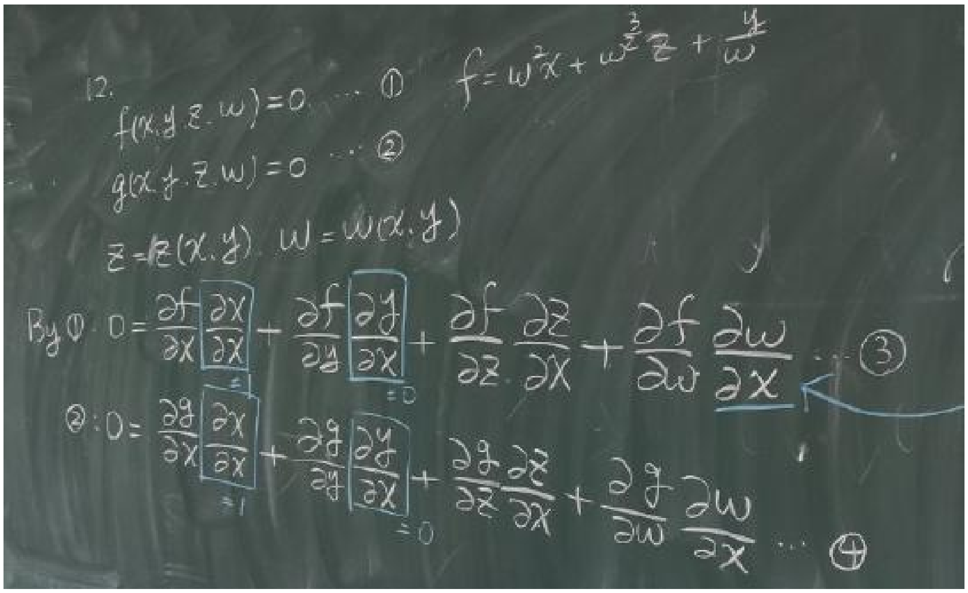
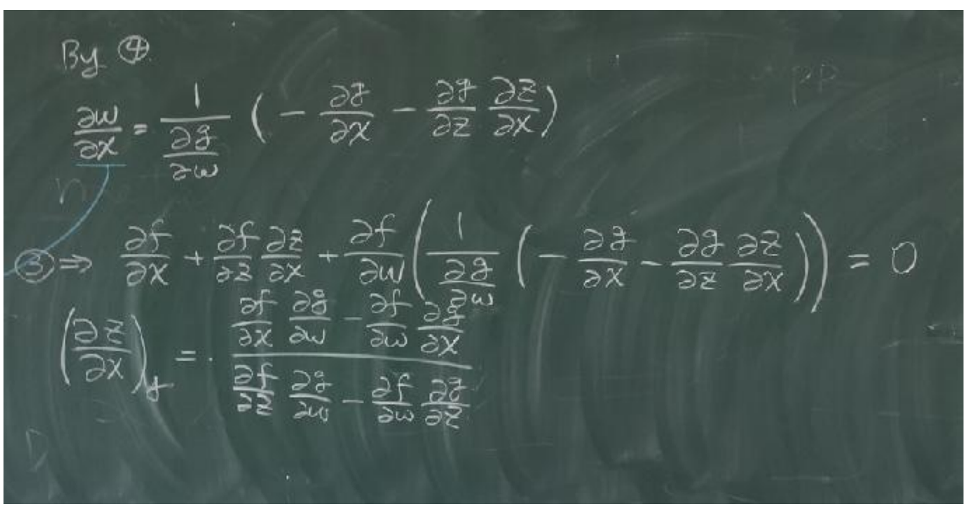
Brief solutions to selected problems in homework week 11

1. Section 14.10, problem 12:





Remark:

$$f_x + f_w w_x + f_z z_x = 0$$

$$g_x + g_w w_x + g_z z_x = 0$$

Two linear equations for the two unknowns z_x and w_x, solve for z_x and w_x.

2. Problem 2:

5 variables (u, x, y, z, w) and 3 (equations+constraints). Therefore 2 independent variables (must be x, y since we want $(\frac{\partial u}{\partial x})_y$.

So $f(x,y,z(x,y),w(x,y))=0$, $g(x,y,z(x,y),w(x,y))=0$ and $u=U(x,y,z(x,y),w(x,y))$.
From the u equation, $u_x = U_x + U_z z_x + U_w w_x$.
From f and g constraints, we have as in problem 12:

$$f_x + f_w w_x + f_z z_x = 0$$

$$g_x + g_w w_x + g_z z_x = 0$$

then solve for w_x and z_x, plug them into $u_x = U_x + U_z z_x + U_w w_x$ to get the desired formula of $(\frac{\partial u}{\partial x})_y$.