

Brief solutions to selected problems in homework week 7

1. Problem 1:

$$o(\Delta x) + o(\Delta y) = o(\sqrt{\Delta x^2 + \Delta y^2})$$

$$\text{or } \varepsilon_1 \Delta x + \varepsilon_2 \Delta y = \varepsilon \sqrt{(\Delta x)^2 + (\Delta y)^2}$$
 part (I):

$$\varepsilon_1 \Delta x + \varepsilon_2 \Delta y = (**) \sqrt{\Delta x^2 + \Delta y^2}$$
 and $\lim(**) = 0$
 Here $(**) = \left(\varepsilon_1 \frac{\Delta x}{\sqrt{\Delta x^2 + \Delta y^2}} + \varepsilon_2 \frac{\Delta y}{\sqrt{\Delta x^2 + \Delta y^2}} \right) \xrightarrow{\Delta x, \Delta y \rightarrow 0} 0$
 part II: Need to decompose $(\Delta x, \Delta y) \rightarrow (0, 0)$

$$\varepsilon \sqrt{\Delta x^2 + \Delta y^2} = \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$$
 "See Hint" $\Rightarrow \varepsilon_1 = \varepsilon \frac{\Delta x}{\sqrt{\Delta x^2 + \Delta y^2}}, \varepsilon_2 = \varepsilon \frac{\Delta y}{\sqrt{\Delta x^2 + \Delta y^2}}$

2. Section 14.4, problem 51:

$$\frac{d}{dx} \underbrace{\int_0^x f(y) dy}_{\text{function of } x} = \frac{d}{dx} (F(x) - F(0)) = f(x)$$

$$\frac{d}{dx} \int_0^1 f(x) dx \rightarrow \frac{d}{dx} \int_0^1 f(y) dy = 0$$

$$\frac{d}{dx} \underbrace{\int_0^1 g(x, y) dy}_{P(x)} = \lim_{\Delta x \rightarrow 0} \int_0^1 \frac{g(x+\Delta x, y) - g(x, y)}{\Delta x} dy$$

$$= \int_0^1 g_x(x, y) dy$$

$$\frac{d}{dx} \underbrace{\int_0^x h(x, y) dy}_{Q(x)} = \frac{d}{dx} \int_0^{s(x)} h(\underline{t(x)}, y) dy \begin{matrix} s(x)=x \\ t(x)=x \end{matrix}$$

$$\frac{d}{dx} W(s(x), t(x))$$

(Q(x))

Figure 1: Section 14.4, problem 51

In general

$$\frac{d}{dx} \int_0^{s(x)} h(t(x), y) dy$$

Eg

$$= \frac{d}{dx} W(s(x), t(x))$$

$$= \partial_1 W(s(x), t(x)) s'(x) + \partial_2 W(s(x), t(x)) \cdot t'(x)$$

$$= h(t(x), s(x)) \cdot s'(x) + \left(\int_0^{s(x)} \partial_1 h(t(x), y) dy \right) t'(x)$$

If $s(x) = x$. $t(x) = x$

$$= h(x, x) + \int_0^x \partial_1 h(x, y) dy$$

Figure 2: Section 14.4, problem 51, continued

$$\begin{aligned}
 \text{Eg } \frac{d}{dx} \int_0^{x^2} \sqrt{t^4 + x^3} dt \\
 &= \sqrt{(x^2)^4 + x^3} \cdot \frac{d}{dx} x^2 \\
 &\quad + \int_0^{x^2} \frac{\partial}{\partial x} \sqrt{t^4 + x^3} dt \\
 &= 2x \sqrt{x^8 + x^3} \\
 &= \frac{3}{2} x^2 \int_0^{x^2} \frac{1}{\sqrt{t^4 + x^3}} dt
 \end{aligned}$$

Figure 3: Section 14.4, problem 51, continued