

Brief solutions to selected problems in homework week 6

1. Section 14.2, problem 51:

14.2 Problem 51

$f(x, y) = 1$

$(0, 1)$

$(2, 3)$

$f(x, y) = 0$

$f(x, y) = 1$

(a) $\lim_{(x, y) \rightarrow (0, 1)} f(x, y) = ?$

Take $\delta = \frac{1}{2}$

$\sqrt{(x-0)^2 + (y-1)^2} < \frac{1}{2}$

$|x| < \frac{1}{2}, y > \frac{1}{2}$

$\Rightarrow x^4 < y$

(b) take $\delta = \frac{1}{2}$

$$\sqrt{(x-2)^2 + (y-3)^2} < \frac{1}{2}$$

$$\frac{3}{2} < x < \frac{5}{2}, \quad \frac{5}{2} < y < \frac{7}{2}$$

$$x^4 > \frac{81}{16} > \frac{7}{2} > y > 0$$

(a), (b) Given $\epsilon > 0$

Take $\delta = \frac{1}{2}$ will do.

(c) Two path Thm

$$\lim_{x \rightarrow 0} f(x, x^6) = 0$$

$$\lim_{x \rightarrow 0} f(x, x^2) = 1$$

Figure 1: Section 14.2, problem 51

2. Section 14.3, problem 60:

60. $f(x,y) = \begin{cases} \frac{\sin(x^3+y^4)}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

$\frac{\partial f}{\partial y} = \lim_{h \rightarrow 0} \frac{f(0,h) - f(0,0)}{h}$

$= \lim_{h \rightarrow 0} \frac{\sin(h^4) - 0}{h^3}$

$= \lim_{h \rightarrow 0} \frac{\sin(h^4)}{h^3} = 1$

$= \lim_{h \rightarrow 0} h \cdot \frac{\sin(h^4)}{h^4}$

$= 0$

Figure 2: Section 14.3, problem 60