

Brief solutions to selected problems in homework week 5

1. Section 10.9, problem 41:

41.
$$e^x = 1 + x + \frac{x^2}{2!} + R_2(x)$$
$$R_2(x) = \frac{f^{(3)}(c)}{3!} (x-0)^3$$
$$|x| < 0.1$$
$$\left| e^x - \left(1 + x + \frac{x^2}{2!} \right) \right| = \left| \frac{e^c}{3!} x^3 \right|$$
$$\left(c \text{ 代 } 0.1, e^c \frac{\text{取大}}{\text{取大}} \right)$$
$$\left(|x| \text{ 代 } 0.1, |x|^3 \frac{\text{取大}}{\text{取大}} \right) \leq \frac{e^{0.1}}{6} (0.1)^3$$

Figure 1: Section 10.9, problem 41

2. Section 10.10, problem 65:

65.

$$\begin{aligned} \sin^{-1} x &= \sin^{-1} 0 + \int_0^x (\sin^{-1} t)' dt \\ &= \int_0^x (1-t^2)^{-\frac{1}{2}} dt \\ &= \int_0^x 1 + \sum_{k=1}^{\infty} \binom{-\frac{1}{2}}{k} (-t^2)^k dt \\ &= \cancel{x} + \sum_{k=1}^{\infty} \binom{-\frac{1}{2}}{k} \frac{(-1)^k \cancel{x}^{2k+1}}{(2k+1)} \end{aligned}$$

Figure 2: Section 10.10, problem 65

3. Section 10.10, problem 65:

$|\frac{1}{t^2}| < 1 \quad |t| > 1$

66. $\tan^{-1} x = \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \dots \quad (x > 1)$

$\frac{1}{1+t^2} = \frac{1}{t^2} \cdot \frac{1}{1+(\frac{1}{t^2})} = \frac{1}{t^2} - \frac{1}{t^4} + \frac{1}{t^6} - \dots$

$\tan^{-1} x = \tan^{-1} b - \int_x^b (\tan^{-1} t)' dt$

$= \lim_{b \rightarrow \infty} \tan^{-1} b - \int_x^{\infty} (\tan^{-1} t)' dt$

$= \frac{\pi}{2} - \int_x^{\infty} \left(\frac{1}{t} - \frac{1}{t^3} + \frac{1}{t^5} - \dots \right) dt$

$= \frac{\pi}{2} - \left(\frac{1}{x} - \frac{1}{3x^3} + \frac{1}{5x^5} - \dots \right)$

Figure 3: Section 10.10, problem 66