

Brief solutions to selected problems in homework week 4

1. Section 10.7, problem 55:

55

$$\begin{aligned} \cos x &= (\sin x)' \\ &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \\ &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \end{aligned}$$

By ratio test $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{x^2}{(2n+1)(2n+2)} \right| \leq 0$

\Rightarrow for every $x \in \mathbb{R}$ the series conv.

b.

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

is also conv. for all x

then let $f(x) = 2x$

$$\sin 2x = \sum_{n=0}^{\infty} (-1)^n \frac{[f(x)]^{2n+1}}{(2n+1)!}$$

is also conv. for all x

conv.

$$= 2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

$$\sin x \cos x$$

$$= \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right) \cdot \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \right)$$

$$= \left(0 + x + 0x^2 - \frac{x^3}{3!} + 0x^4 + \frac{x^5}{5!} - \dots \right)$$

$$\cdot \left(1 + 0x - \frac{x^2}{2!} + 0x^3 + \frac{x^4}{4!} - 0x^5 + \dots \right)$$

$$= \underline{1x - \frac{2}{3}x^3 + \frac{2}{15}x^5 - \dots}$$

$$x^0: 0 \cdot 1 = 0$$

$$x^1: 1 \cdot 1 + 0 \cdot 0 = 1$$

$$x^2: 0 \cdot \left(\frac{1}{2}\right) + 1 \cdot 0 + 0 \cdot 1 = 0$$

$$x^3: 0 \cdot 0 + 1 \cdot \left(-\frac{1}{2!}\right) + 0 \cdot 0 + \left(\frac{1}{3!}\right) \cdot 1 = -\frac{2}{3}$$

$$x^4: 0 \cdot \frac{1}{4!} + 1 \cdot 0 + 0 \cdot \left(\frac{1}{2!}\right) + \left(\frac{1}{3!}\right) \cdot 0 + 0 \cdot 1 = 0$$

$$x^5: 0 \cdot 0 + 1 \cdot \frac{1}{4!} + 0 \cdot 0 + \left(\frac{1}{3!}\right) \cdot \left(-\frac{1}{2!}\right) + 0 \cdot 0 + \frac{1}{5!} \cdot 1 = \frac{2}{15}$$

Figure 1: Section 10.7, problem 55