

Brief solutions to selected problems in homework week 03

1. Section 10.5, problem 41: No conclusion from ratio test. Use direct comparison test instead.

$$41. \sum_{n=1}^{\infty} \frac{n! \ln n}{n(n+2)!}$$
$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)! \ln(n+1)}{(n+1)(n+3)!} \cdot \frac{n(n+2)!}{n! \ln n} \right|$$
$$= \lim_{n \rightarrow \infty} \left| \frac{\ln(n+1)}{n+3} \cdot \frac{n}{\ln n} \right| = 1$$

no conclusion

$$n(n+1)(n+2) > n^3$$
$$\Rightarrow \frac{1}{n^3} > \frac{1}{n(n+1)(n+2)}$$
$$0 < \frac{n! \ln n}{n(n+2)!} = \frac{\ln n}{n(n+1)(n+2)} \leq \frac{\ln n}{n^3}$$

$< \frac{1}{n^2}$, convergent by direct comparison

Figure 1: Section 10.5, problem 41