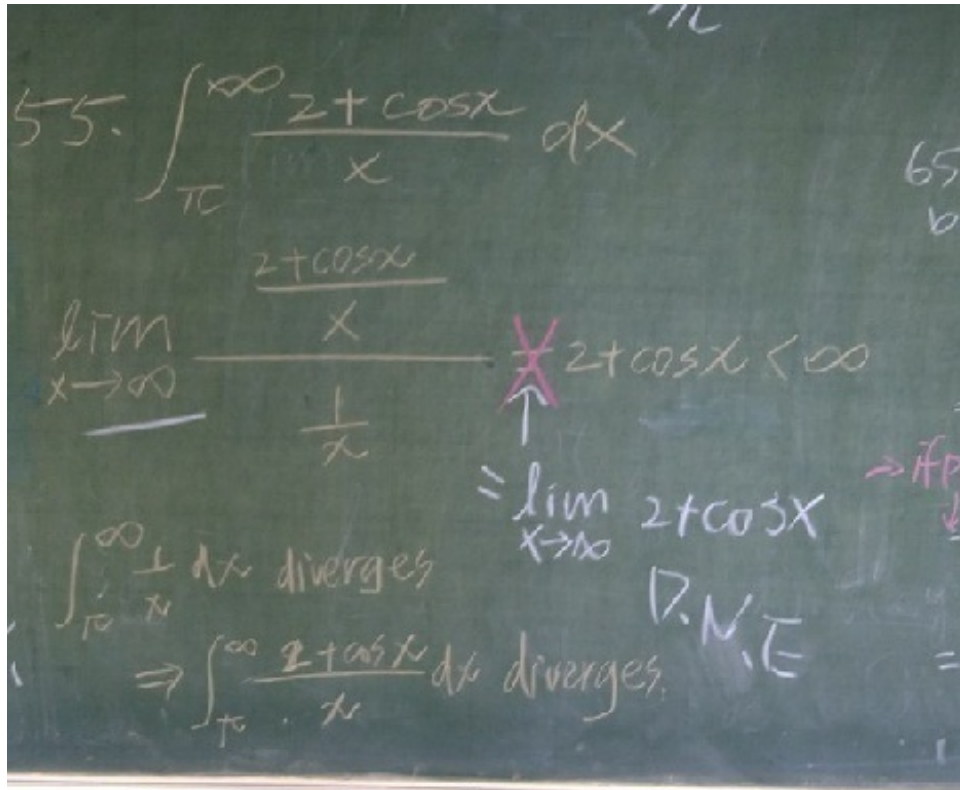


Brief solutions to selected problems in homework week 01

1. Section 8.8, problem 55:



Can not use limit comparison.

Use direct comparison:

$$\frac{1}{x} \leq \frac{2 + \cos x}{x}$$

ans: divergent.

Figure 1: Section 8.8, problem 55

2. Section 8.8, problems 81,82, method 1: (method 2 is $x = e^y$ and integration by parts)

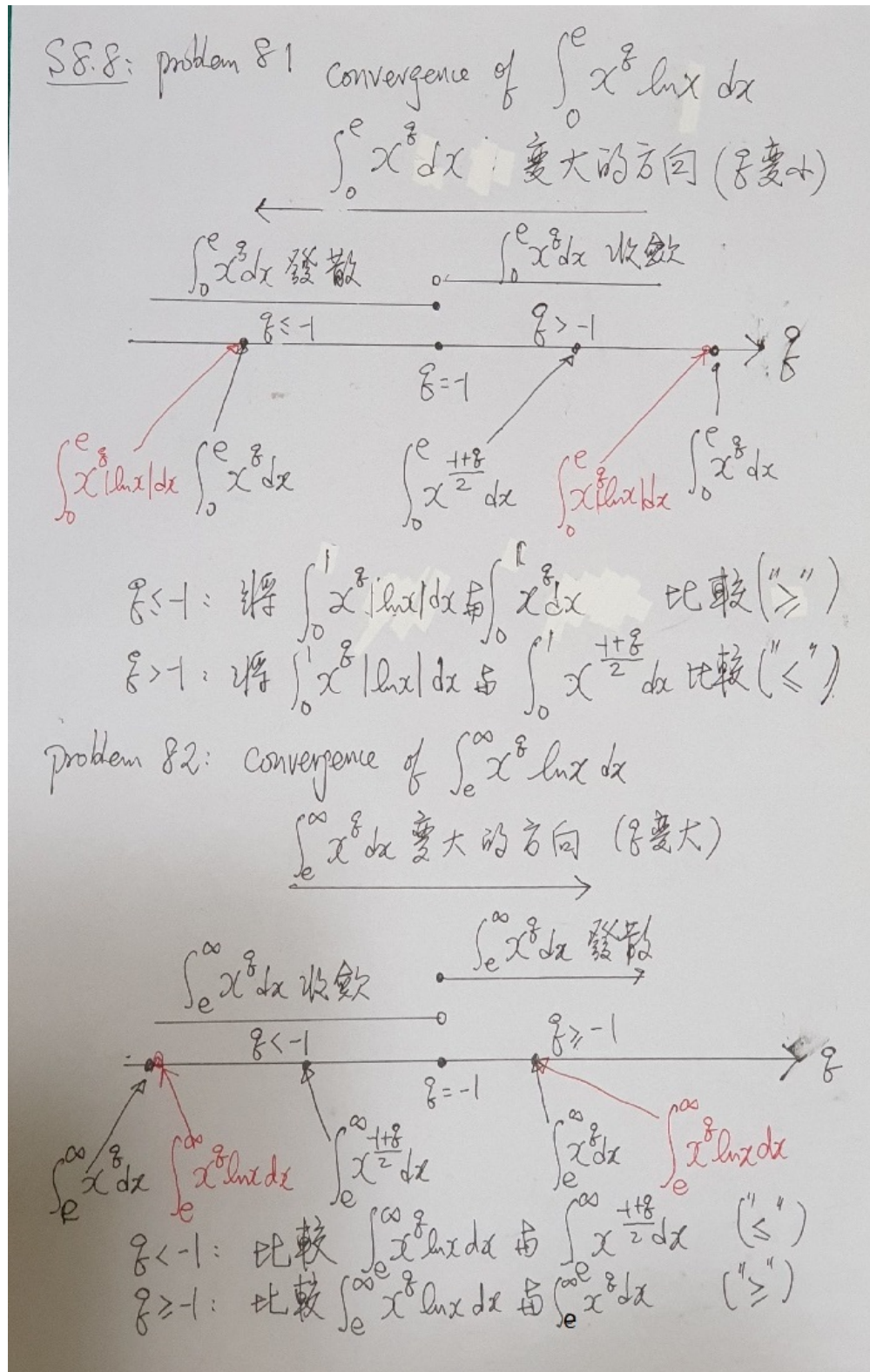


Figure 2: Section 8.8, problems 81,82

Problem 83: From problem 81 and problem 82, $\int_0^\infty x^q \ln x dx$ diverges for all $q \in \mathbb{R}$.

3. Problem 3:

3. $\int_0^1 \cot^p x dx$

$$\lim_{x \rightarrow 0^+} \frac{x^{-p}}{\cot^p x}$$
$$= \lim_{x \rightarrow 0^+} \frac{\sin^p x}{x^p \cos^p x}$$

= | so, same as $\int_0^1 x^{-p} dx$

Converges for $0 < p < 1$ and
diverges for $p \geq 1$.

Figure 3: Problem 3

4. Chapter 8, Additional and advanced problems, problem 8:

$$\lim_{x \rightarrow 0^+} x \int_x^1 \frac{\cos t}{t^2} dt$$

$$= \lim_{x \rightarrow 0^+} \frac{-\int_1^x \frac{\cos t}{t^2} dt}{\frac{1}{x}}$$

$$= \lim_{x \rightarrow 0^+} \frac{-\frac{\cos x}{x^2}}{-\frac{1}{x^2}}$$

$$= 1$$

$$\Rightarrow 0 \leq \frac{\cos t}{t^2} \leq \frac{\cos t}{t^2}$$

$$\cos t \int_0^1 \frac{1}{t^2} dt \text{ div}$$

By $\int_x^1 \frac{\cos t}{t^2} dt \text{ div}$

$$0 < \frac{1}{x}$$

$$t \in (0, 1]$$

$$0 \leq \cos t \leq \cos t$$

$$\Rightarrow 0 \leq \frac{\cos t}{t^2} \leq \frac{\cos t}{t^2}$$

$$\cos t \int_0^1 \frac{1}{t^2} dt \text{ div}$$

By $\int_x^1 \frac{\cos t}{t^2} dt \text{ div}$

Above shows the improper integral diverges, so limit on the left is of the type infinity/infinity

Figure 4: Chapter 8, Additional and advanced problems, problem 8

5. Section 10.1, problem 63:

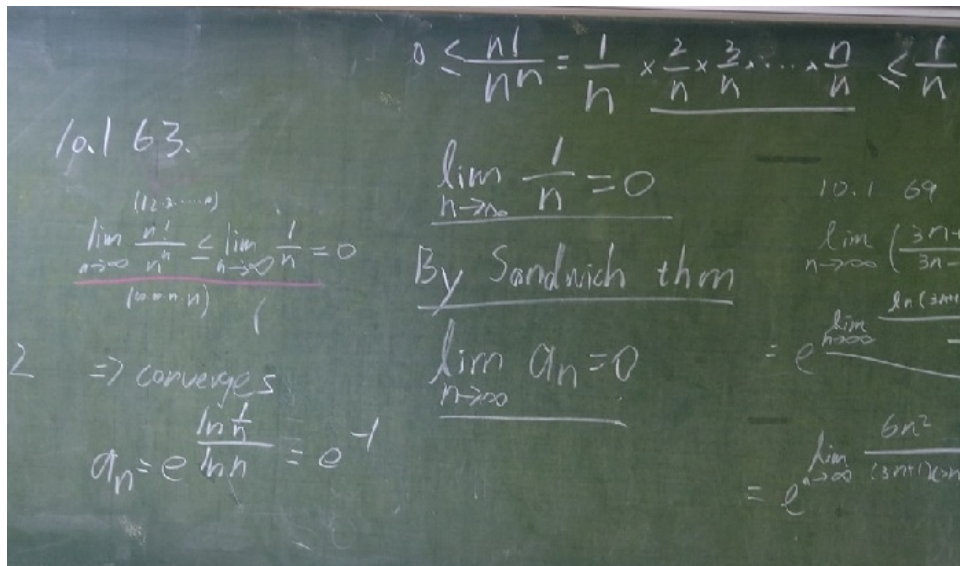


Figure 5: Section 10.1, problem 63