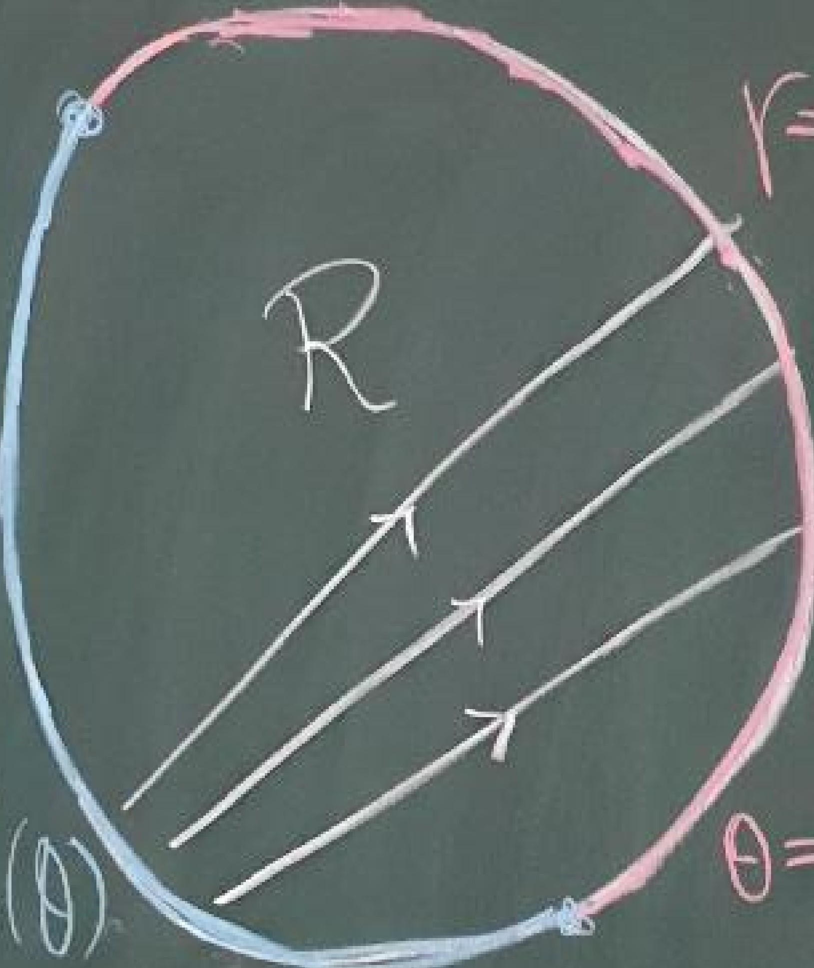
Double integral in Polar Coord.

(i) $dA = r\,dr\,d\theta$

(ii)

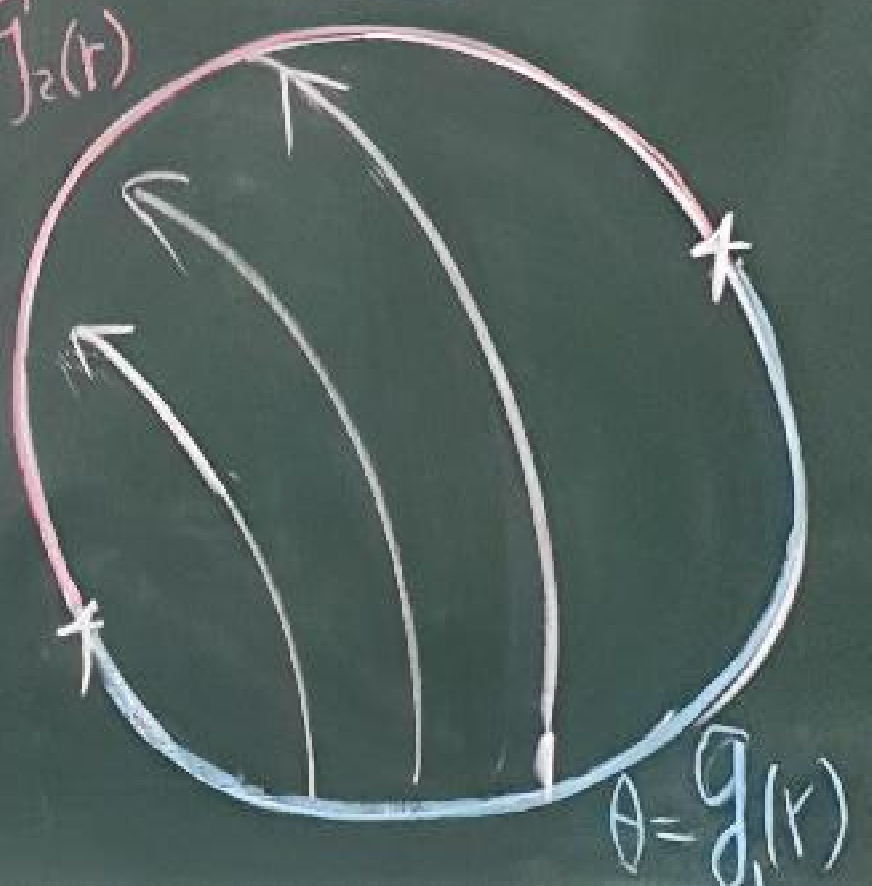


Find $f_1(\theta), f_2(\theta)$

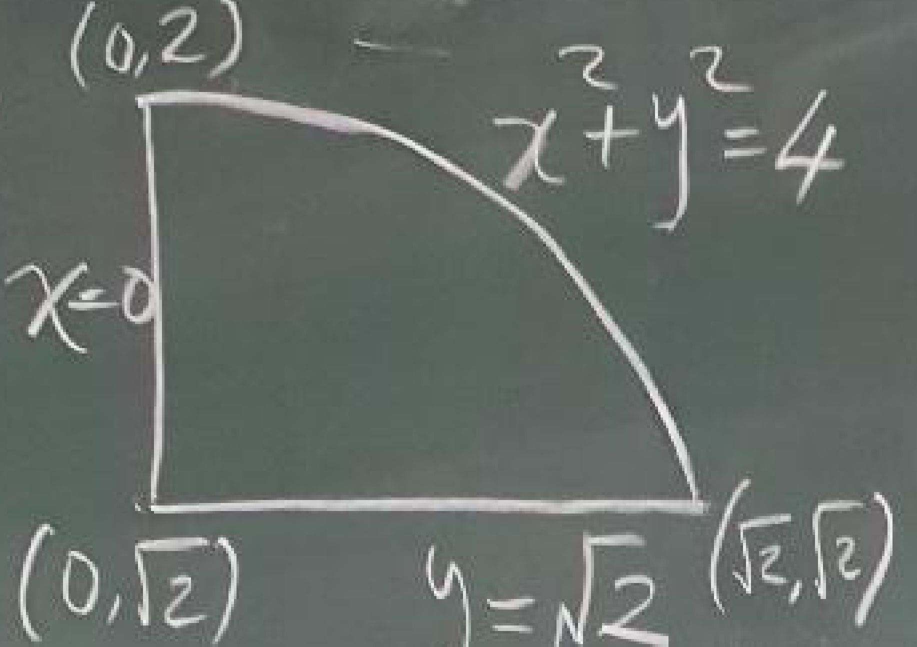
$$I=\int_{\theta_1}^{\theta_2}\int_{f_1(\theta)}^{f_2(\theta)} (*)\, r\,dr\,d\theta$$

or Find $g_1(r), g_2(r)$

$$I=\int_{r_1}^{r_2}\int_{g_1(r)}^{g_2(r)} (*)\, d\theta\, r\,dr$$



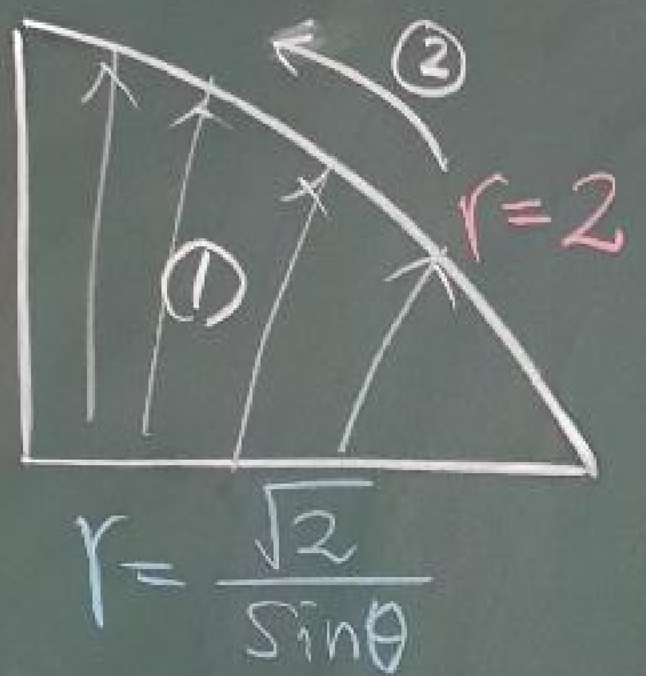
Eg



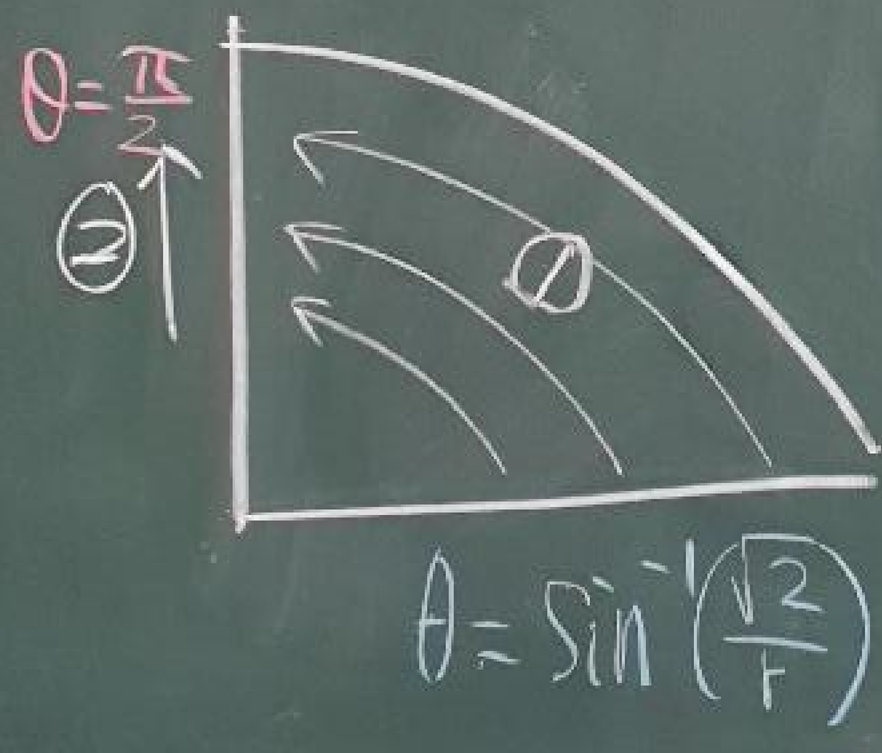
$$x^2+y^2=4 \Leftrightarrow r=2$$

$$x=0 \Leftrightarrow r\cos\theta=0$$

$$y=\sqrt{2} \Leftrightarrow r\sin\theta=\sqrt{2}$$

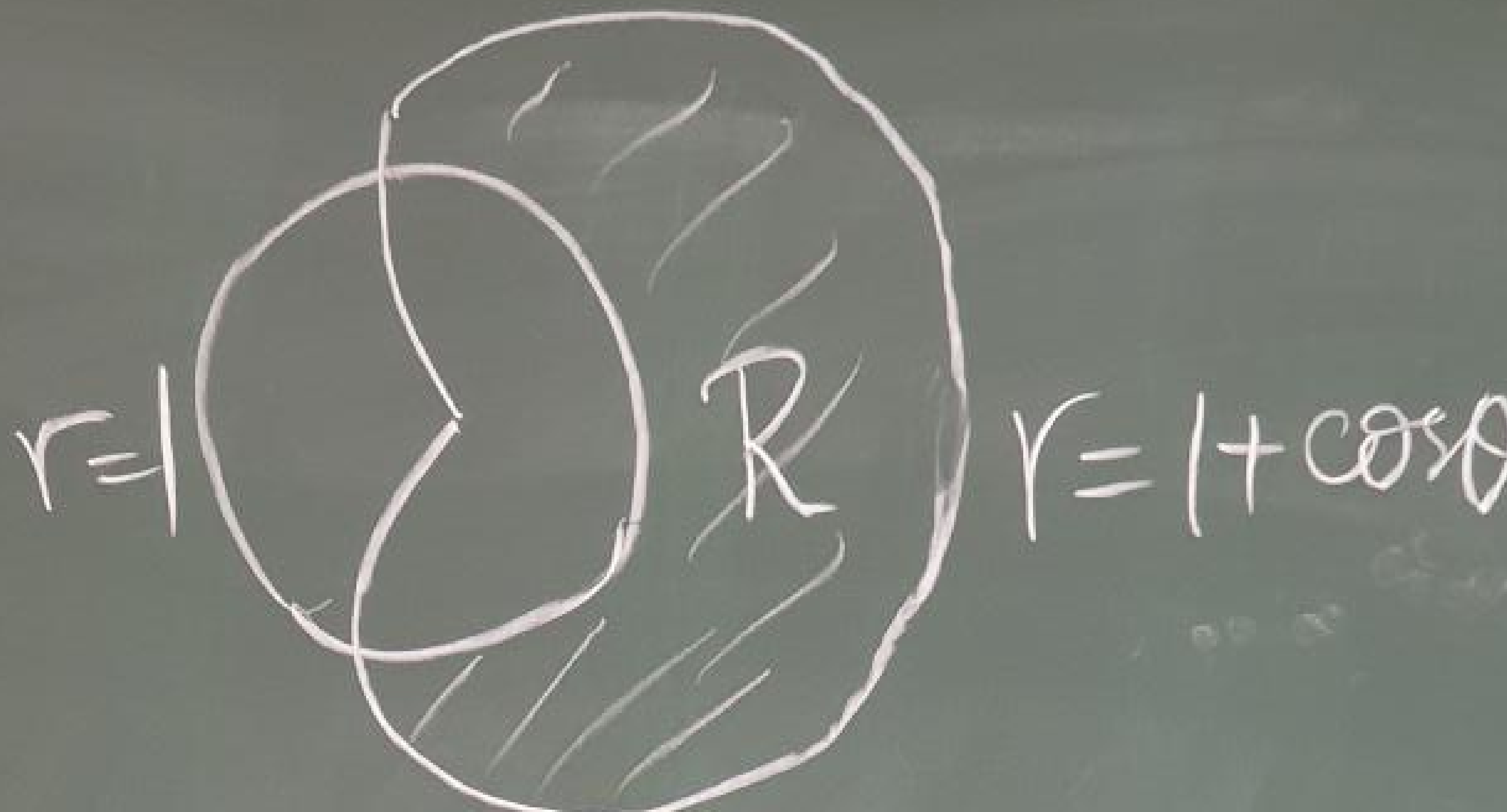


$$\Rightarrow \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{\frac{\sqrt{2}}{\sin\theta}}^{2} (*)\, r\,dr\,d\theta$$

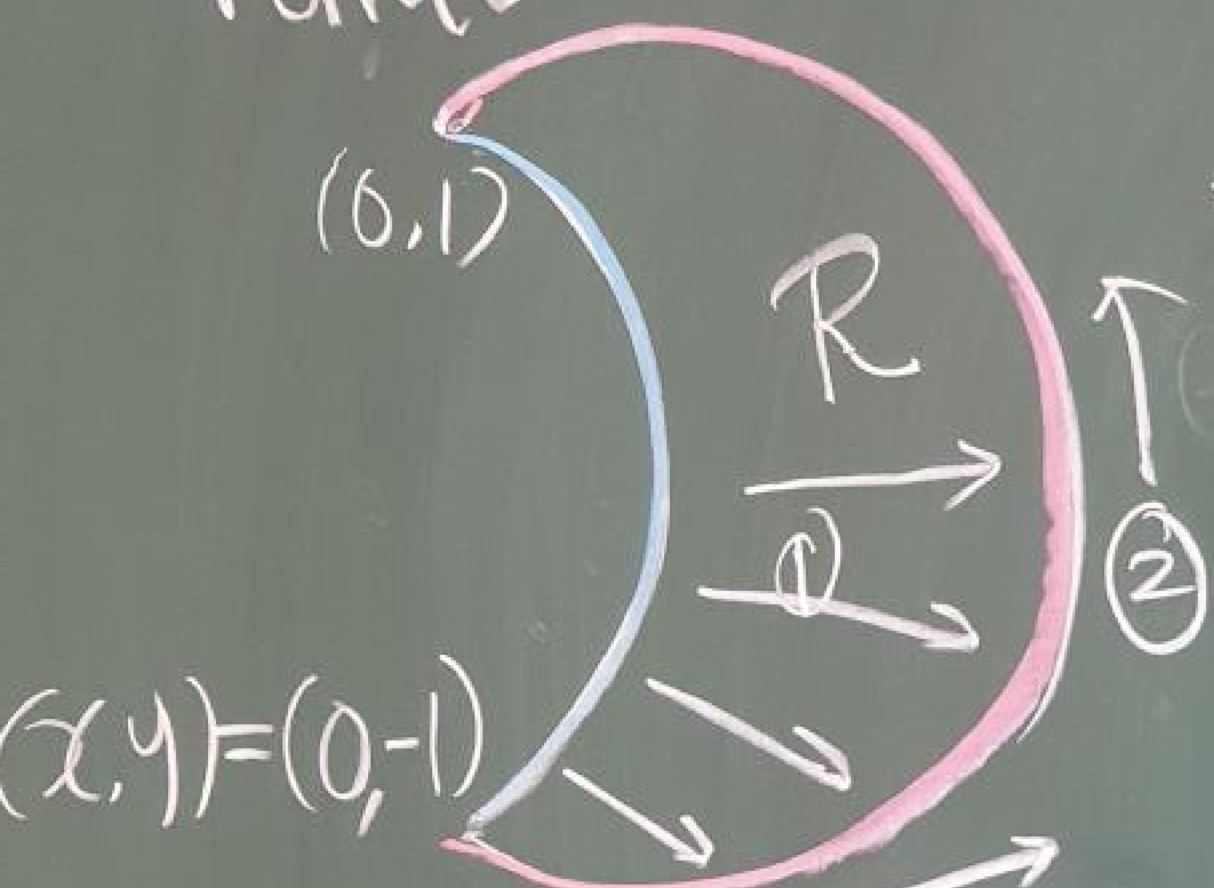


$$\Rightarrow \int_{\sqrt{2}}^{2} \int_{\theta=\sin^{-1}\left(\frac{\sqrt{2}}{r}\right)}^{\frac{\pi}{2}} (*)\, d\theta\, r\,dr$$

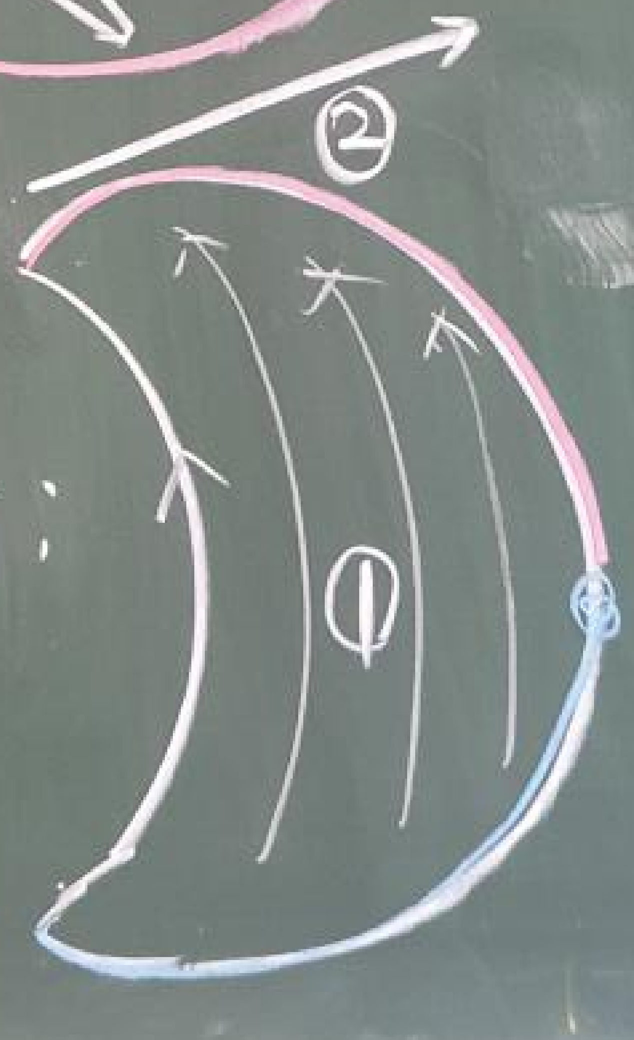
Eg



rdrdθ



$$I=\int_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{r=1}^{1+\cos\theta}(*)\,r\,dr\,d\theta$$



dθ rdr:

$$\theta=\cos^{-1}(r-1)$$

$$\theta=-\cos^{-1}(r-1)$$

$$I=\int_{r=1}^{2}\int_{-\cos^{-1}(r-1)}^{\cos^{-1}(r-1)}(*)\,d\theta\,r\,dr$$

Rm $r\,dr\,d\theta$ is usually preferred choice since curves in polar coordinates are generally expressed as $r = f(\theta) \cdots (*)$

$$\int_{\theta_1}^{\theta_2}\int_{(*)}^{(*)} r\,dr\,d\theta$$

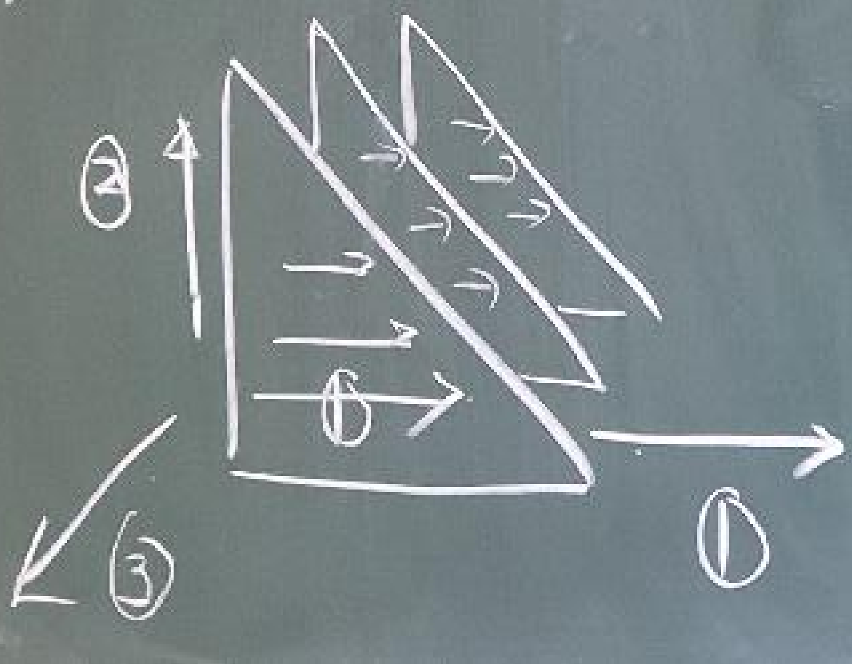
Triple integral in Cartesian Coord.

$$I = \iiint_D f(x,y,z)\,dV = \lim_{\|P\|\to 0} \sum_{k=1}^{n} f(x_k, y_k, z_k)\,\Delta V$$

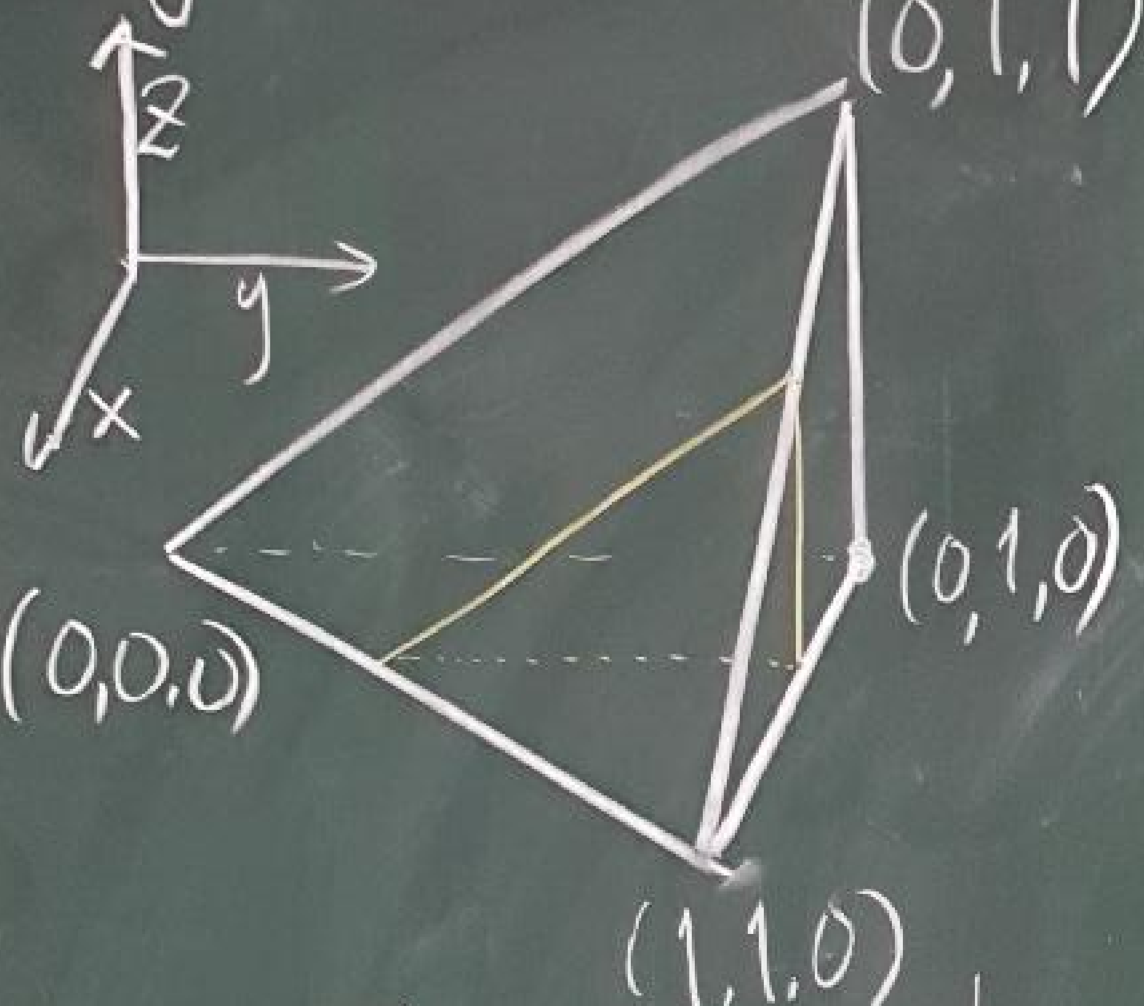
When $f(x,y,z) = 1$, $I =$ volume of D

(i) In cartesian coord. $dV = dx\,dy\,dz$

(ii) $d① \, d② \, d③$



Eg Volume = ?



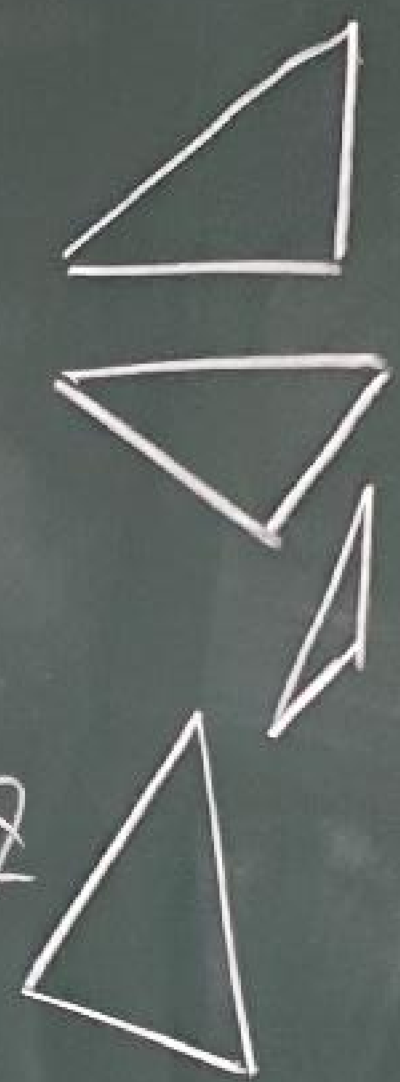
D = domain bounded by

$x=0$ (A

$z=0$ (B

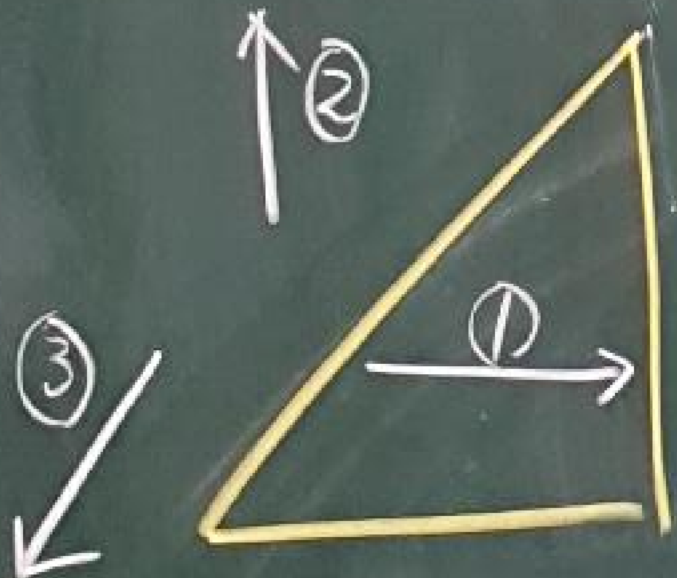
$y=1$ (C

$y=x+z$ (D



Case 1 : $dy\,dz\,dx$

Step 1.1 fix x, sketch y,z part



Step 1.2: fix z, x

①: $= \int_{(D)}^{(C)} dy$

(C): $y=1$

(D): $y=x+z$

$= \int_{x+z}^{1} dy$

Step2 $\displaystyle\int_{(B)}^{(C\cap D)}\int_{x+z}^{1} 1\,dy\,dz$

(B): $z=0$

$(C\cap D)$: $z=$ function of x
$= 1-x$

$$①+② = \int_{z=0}^{1-x}\int_{y=x+z}^{1} 1\,dy\,dz$$

Step 3 ③: $\displaystyle\int_{x=0}^{1} dx$

$$V = \int_{x=0}^{1}\int_{z=0}^{1-x}\int_{y=x+z}^{1} 1\,dy\,dz\,dx$$