Homework Assignment for Week 15

1. Section 16.3:
   Let $\boldsymbol{F} = \dfrac{x}{\sqrt{x^2+y^2}}\,\boldsymbol{i} + \dfrac{y}{\sqrt{x^2+y^2}}\,\boldsymbol{j} + 0\,\boldsymbol{k}$ and $\boldsymbol{G} = \dfrac{-y}{x^2+y^2}\,\boldsymbol{i} + \dfrac{x}{x^2+y^2}\,\boldsymbol{j} + 0\,\boldsymbol{k}$.
   (a) Show that both $\boldsymbol{F}$ and $\boldsymbol{G}$ satisfy the component test.
   (b) The natural domain of both $\boldsymbol{F}$ and $\boldsymbol{G}$ is $\{(x,y,z),\ x^2+y^2 \neq 0\}$ (that is where $\boldsymbol{F}$ and $\boldsymbol{G}$ are defined). Show that $\boldsymbol{F}$ is conservative in this domain by finding its potential function.
   (c) Show that $\boldsymbol{G}$ is NOT conservative in this domain (see Example 5 on p990).
   (d) If given another $\boldsymbol{H}$ satisfying the component test in this domain, how do you determine whether $\boldsymbol{H}$ is conservative?

2. Section 16.3:
   Let $\boldsymbol{F} = \dfrac{x}{\sqrt{x^2+y^2+z^2}}\,\boldsymbol{i} + \dfrac{y}{\sqrt{x^2+y^2+z^2}}\,\boldsymbol{j} + \dfrac{z}{\sqrt{x^2+y^2+z^2}}\,\boldsymbol{k}$. What is the natural domain of $\boldsymbol{F}$? Show that $\boldsymbol{F}$ satisfies the component test in this domain. Is this domain simply connected? Is $\boldsymbol{F}$ conservative in this domain?

3. Section 16.4: Problems 10, 17, 19, 23, 27, 38, 39.

   Hints:

   In problem 17, a polar curve $r = f(\theta)$ can be parameterized by $x(\theta) = f(\theta)\cos\theta$, $y(\theta) = f(\theta)\sin\theta$.

   Problem 19 can be computed easier using Green's Theorem.

   On a circle $x^2+y^2=a^2$, it is a useful tip to remember that $\boldsymbol{n} = \dfrac{(x,y)}{a}$, $ds = a\,d\theta$ and $\boldsymbol{n}\,ds = (x,y)\,d\theta$.

4. Section 16.4: Verify Green's Theorem in tangential form and normal form for the vector field $\boldsymbol{F} = (M(x,y), 0)$ on R, where M and its partial derivatives are all continuous in R, the region illustrated in class. That is, R is bounded by $x=0$, $y=0$ and the curve $y=f(x)$, $0 \le x \le a$ with $f(0)=b$ and $f(a)=0$, which at the same time can be described as $x=g(y)$, $0 \le y \le b$ with $g(0)=a$ and $g(b)=0$. In the computation of the line integrals on the three portions of ∂R, pay attention to finding suitable parametrizations with correct orientation.

5. Section 16.5: Problems 5, 11, 13, 19, 31, 33, 49, 51, 55, 56.

   Hints:

Problems 5-19: Among other choices, most of the parametrizations can be chosen as $x(r, \theta) = r\cos\theta$, $y(r, \theta) = r\sin\theta$ and $z(r, \theta) = f(x(r, \theta), y(r, \theta))$, or $\boldsymbol{r} = \boldsymbol{r}(\theta, \phi)$ as in spherical coordinate with ρ fixed as a constant.

Problem 51: Read Example 8 and show that $d\sigma = \sqrt{2}dA$. A similar (slightly different) formula also applies to problem 49.