

Homework Assignment for Week 14

1. Section 16.1: Problems 15, 23, 25, 29.

2. Section 16.2: Problems 19, 25, 29(b), 35(a).

See equation (5) in p978 and (6) in p979 for definition of flow, circulation and flux.

3. Section 16.3: Problems 1, 3, 5, 9, 21, 26, 29.

Hint: For problems 1, 3, 5, use Component Test on p988. For problem 29: do not evaluate the line integrals directly. Try to find the potential function f for the vector field \mathbf{F} instead.

4. Section 16.3: (homework problem for next week)

$$\text{Let } \mathbf{F} = \frac{x}{\sqrt{x^2 + y^2}}\mathbf{i} + \frac{y}{\sqrt{x^2 + y^2}}\mathbf{j} + 0\mathbf{k} \text{ and } \mathbf{G} = \frac{-y}{x^2 + y^2}\mathbf{i} + \frac{x}{x^2 + y^2}\mathbf{j} + 0\mathbf{k}.$$

(a) Show that both \mathbf{F} and \mathbf{G} satisfy the component test.

(b) The natural domain of both \mathbf{F} and \mathbf{G} is $\{(x, y, z), x^2 + y^2 \neq 0\}$ (that is where \mathbf{F} and \mathbf{G} are defined). Show that \mathbf{F} is conservative in this domain by finding its potential function.

(c) Show that \mathbf{G} is NOT conservative in this domain (see Example 5 on p990).

(d) If given another \mathbf{H} satisfying the component test in this domain, how do you determine whether \mathbf{H} is conservative?

5. Section 16.3: (homework problem for next week)

Let $\mathbf{F} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}\mathbf{i} + \frac{y}{\sqrt{x^2 + y^2 + z^2}}\mathbf{j} + \frac{z}{\sqrt{x^2 + y^2 + z^2}}\mathbf{k}$. What is the natural domain of \mathbf{F} ? Show that \mathbf{F} satisfies the component test in this domain. Is this domain simply connected? Is \mathbf{F} conservative in this domain?