Homework Assignment for Week 07

1. Section 14.3: Show that if $g(x,y) = \varepsilon_1 \cdot (x - x_0) + \varepsilon_2 \cdot (y - y_0)$ as $(x,y) \to (x_0,y_0)$ then $g(x,y) = \varepsilon \cdot \sqrt{(x-x_0)^2 + (y-y_0)^2}$ and vice versa (the converse).
   Here $\lim_{(x,y)\to(x_0,y_0)} (\varepsilon_1, \varepsilon_2, \varepsilon) = (0,0,0)$.

   Remark: An alternative expression of the above statement reads:

   Show that
   $$o(1) \cdot \Delta x + o(1) \cdot \Delta y = o(1) \cdot \sqrt{\Delta x^2 + \Delta y^2},$$
   or
   $$o(\Delta x) + o(\Delta y) = o(\sqrt{\Delta x^2 + \Delta y^2}),$$
   where the $o(\cdot)$'s refer to $2D$ limits as $(\Delta x, \Delta y) \to (0,0)$.

   Hint: $\sqrt{\Delta x^2 + \Delta y^2} = \dfrac{\Delta x}{\sqrt{\Delta x^2 + \Delta y^2}} \Delta x + \dfrac{\Delta y}{\sqrt{\Delta x^2 + \Delta y^2}} \Delta y$

2. Section 14.4: Problems 1, 7, 10, 21, 24, 29, 31, 43, 51.

3. Section 14.4: Suppose that $F(x,y,z) = 0$ can implicitly define $x = f(y,z)$, or $y = g(z,x)$, or $z = h(x,y)$ near some point (x_0,y_0,z_0) with $F(x_0,y_0,z_0) = 0$. (for example, $F(x,y,z) = x + 2y + 3z - 4$ can). Show that, for any such point (x_0,y_0,z_0), we have
   $$\frac{\partial f}{\partial y}\frac{\partial g}{\partial z}\frac{\partial h}{\partial x} = \frac{\partial f}{\partial z}\frac{\partial g}{\partial x}\frac{\partial h}{\partial y} = -1$$

   Hint: Read Example 5 and Example 6.

4. Section 14.5: (homework problems for next week) Problems 9, 15, 19, 25, 27, 29, 35, 36, 40 (See page 850).