Homework Assignment for Week 04

1. Section 10.7: 29, 33, 37, 40, 43, 47, 51, 55, 60.

   Hint for problem 51: $x-2=x-5+3=3(1+\frac{x-5}{3})$.

   Remark on problem 55: Take the sentence "converges to $\sin x$" for granted for now. part (c): first six terms means coefficients of $1, x, \cdots, x^5$.

2. Section 10.7: Find a power series that converges on $(1,3)$ and diverges otherwise. Do the same for $(1,3]$, $[1,3)$ and $[1,3]$, respectively.

3. Section 10.8: Problems 5, 7, 15, 23, 29, 35.

   Remark for problem 5, 7: Taylor polynomial of order n generated by f at a is simply the first $n+1$ terms of the Taylor series generated by f at a. See the definition on page 642. Do problem 5 and 7 for $n=3$.

   Hint for problem 5: Another way of finding the answer (without performing all the derivatives) is to write $x=x-2+2=2(1+\frac{x-2}{2})$ and the formula for geometric series.

   Remark for problem 23: We know that $f(x)=\sum_{n=0}^{3} b_n(x-2)^n$ for some c_n's (for example, one can conclude this by repeated division by $(x-2)$). Nevertheless, it is enough to assume $f(x)$ can be written in this form. Using a similar procedure as in page 6 of Lecture 08 note, one can show that the final answer is the same as $f(x)$ without explicitly knowing the values of c_n.

4. Section 10.8: Let

$$f(x)=\begin{cases} 0, & x=0 \\ e^{-1/x^2}, & x\neq 0 \end{cases}$$

   It is known that $f^{(n)}(0)=0$ for all n. Verify this for $f'(0)$ and $f''(0)$ (go through the details).