

Homework Assignment for Chapter 14

1. Section 14.2: Problems 41, 43, 48, 49, 51, 57, 61, 63, 67.

Hint for problems 61, 63: Read ‘Changing to Polar Coordinates’ on page 781. Note that in equation (1), page 781, “ $|r| < \delta$ ” should be “ $0 < |r| < \delta$ ” instead.

Hint for problem 49: Find two paths that pass through $(1, 1)$.

2. Section 14.3: Problems 19, 21, 53, 60, 65, 67, 69, 81, 91.

3. Show that if $g(x, y) = \varepsilon_1 \cdot (x - x_0) + \varepsilon_2 \cdot (y - y_0)$ as $(x, y) \rightarrow (x_0, y_0)$ then $g(x, y) = \varepsilon \cdot \sqrt{(x - x_0)^2 + (y - y_0)^2}$ and vice versa (the converse). Here $\lim_{(x,y) \rightarrow (x_0,y_0)} (\varepsilon_1, \varepsilon_2, \varepsilon) = (0, 0, 0)$.

Remark: A brief version of the above statement reads:

Show that $o(1) \cdot \Delta x + o(1) \cdot \Delta y = o(1) \cdot \sqrt{\Delta x^2 + \Delta y^2}$ where the $o(1)$'s refer to 2D limits as $(\Delta x, \Delta y) \rightarrow (0, 0)$.

Hint: $\sqrt{\Delta x^2 + \Delta y^2} = \frac{\Delta x}{\sqrt{\Delta x^2 + \Delta y^2}} \Delta x + \frac{\Delta y}{\sqrt{\Delta x^2 + \Delta y^2}} \Delta y$

4. Section 14.4: Problems 1, 7, 10, 21, 24, 29, 31, 43, 51.

5. Suppose that $F(x, y, z) = 0$ can implicitly define $x = f(y, z)$, or $y = g(z, x)$, or $z = h(x, y)$ near some point (x_0, y_0, z_0) with $F(x_0, y_0, z_0) = 0$. (for example, $F(x, y, z) = x + 2y + 3z - 4$ can). Show that, for any such point (x_0, y_0, z_0) , we have

$$\frac{\partial f}{\partial y} \frac{\partial g}{\partial z} \frac{\partial h}{\partial x} = \frac{\partial f}{\partial z} \frac{\partial g}{\partial x} \frac{\partial h}{\partial y} = -1$$

6. Section 14.5: Problems 9, 15, 19, 25, 27, 29, 35, 36, 40 (See page 807).

7. Let $f(x, y) = x^2 y / (x^2 + y^2)$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$.

(a) Is f continuous at $(0, 0)$?

(b) Do f_x and f_y exist at $(0, 0)$?

(c) Evaluate the directional derivative of f at $(x_0, y_0) = (0, 0)$ in the direction $(\cos \theta, \sin \theta)$, i.e. $df/ds_{(0,0),(\cos \theta, \sin \theta)}$, if it exists.

(d) Is f differentiable at $(0, 0)$?

8. Section 14.6: Problems 1, 9, 13, 19, 25, 33, 39, 45, 53, 55, 58, 57.

9. Section 14.7: Problems 1, 19, 31, 35, 39, 43, 44, 49, 51, 55, 61.

Hint for problem 44: Do this problem using the gradient analysis. That is, plot ∇f near the critical point to determine whether the critical point is a local minimum, local maximum or neither.

10. Section 14.8: Problems 1, 23, 27, 37, 39.

11. Section 14.9: Problems 7, 9, 11.

12. Taylor's formula for functions of 2 variables can be summarized as

$$f(x_0 + \Delta x, y_0 + \Delta y) = f(x_0, y_0) + \sum_{k=1}^n \frac{1}{k!} (\Delta x \partial_x + \Delta y \partial_y)^k f(x_0, y_0) \\ + \frac{1}{(n+1)!} (\Delta x \partial_x + \Delta y \partial_y)^{n+1} f(x_0 + c\Delta x, y_0 + c\Delta y), \quad 0 < c < 1.$$

Derive a similar formula for functions of 3 variables.

13. Section 14.10: Problems 3, 7, 9, 12.

14. Continue on problem 12: Give a formula for $\left(\frac{\partial u}{\partial x}\right)_y$ where $u = u(x, y, z, w)$ and $f(x, y, z, w) = 0$, $g(x, y, z, w) = 0$.

15. Follow up on example 5 of section 14.8: Let $f(x, y, z) = x^2 + y^2 + z^2$ and compute $\frac{df}{dx}$ at $P_1 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1 - \sqrt{2}\right)$, subject to the two constraints $g_1(x, y, z) = x^2 + y^2 - 1 = 0$ and $g_2(x, y, z) = x + y + z - 1 = 0$. That is, evaluate $\frac{d}{dx} f(x, y(x), z(x))$ at $x = \frac{\sqrt{2}}{2}$, where $y = y(x)$ and $z = z(x)$ are implicitly given by the constraints $g_1 = 0$ and $g_2 = 0$. Can you explain why you get $\frac{df}{dx} = 0$ there?