4. $\frac{d}{dx}\begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix}$

$= \frac{d}{dx}(f_{11}(x)f_{22}(x) - f_{12}(x)f_{21}(x))$

$= f_{11}'(x)f_{22}(x) + f_{11}(x)f_{22}'(x) - f_{12}'(x)f_{21}(x) - f_{12}(x)f_{21}'(x)$

$= \begin{vmatrix} f_{11}'(x) & f_{12}'(x) \\ f_{21}(x) & f_{22}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}(x) \\ f_{21}(x)' & f_{22}'(x) \end{vmatrix}$

$= \begin{vmatrix} f_{11}'(x) & f_{12}(x) \\ f_{21}'(x) & f_{22}(x) \end{vmatrix} + \begin{vmatrix} f_{11}(x) & f_{12}'(x) \\ f_{21}(x) & f_{22}'(x) \end{vmatrix}$

5. $\frac{d}{dx}(u(x)v(x)) = uv' + u'v$

$\frac{d^2}{dx^2}(u(x)v(x)) = uv'' + u'v' + u'v' + u''v = uv'' + 2u'v' + u''v$

$\frac{d^3}{dx^3}(u(x)v(x)) = uv''' + 3u'v'' + 3u''v' + u'''v$

Claim $\frac{d^n}{dx^n} = \sum_{k=0}^{n}\binom{n}{k}u^{(k)}v^{(n-k)}$

$n=2$ hold, Assume $n=m$ hold

For $n=m+1$, $\frac{d^{m+1}}{dx^{m+1}}(u(x)v(x)) = \frac{d}{dx}\left(\frac{d^m}{dx^m}(u(x)v(x))\right)$

$= \frac{d}{dx}\left(\sum_{k=0}^{m}\binom{m}{k}u^{(k)}v^{(m-k)}\right)$

$= \sum_{k=0}^{m}\binom{m}{k}u^{(k)}v^{(m-k+1)} + \sum_{k=0}^{m}\binom{m}{k}u^{(k+1)}v^{(m-k)} = uv^{(m+1)} + \sum_{k=1}^{m}\Big[\underbrace{\binom{m}{k}+\binom{m}{k-1}}_{=\binom{m+1}{k}}\Big]u^{(k)}v^{(m-k+1)} + u^{(m+1)}v$

$= \sum_{k=0}^{m+1}\binom{m+1}{k}u^{(k)}v^{(m+1-k)}$ hold by Induction, Claim hold ※