

HW W3 = 1 If $\lim_{x \rightarrow c} f(x) = L$, $\lim_{x \rightarrow c} g(x) = m$.

For any $\varepsilon > 0$, there exists $\delta_1 > 0$ and $\delta_2 > 0$, such that

$$0 < |x - c| < \delta_1 \Rightarrow -\frac{\varepsilon}{2} < f(x) - L < \frac{\varepsilon}{2}$$

$$0 < |x - c| < \delta_2 \Rightarrow -\frac{\varepsilon}{2} < g(x) - m < \frac{\varepsilon}{2}$$

$$\Rightarrow \text{take } \delta = \min(\delta_1, \delta_2)$$

$$\Rightarrow \begin{cases} -2\varepsilon < 4f(x) - 4L < 2\varepsilon \\ -\varepsilon < 2g(x) - 2m < \varepsilon \end{cases} \quad \text{X} \quad -\varepsilon < 4f(x) - 2g(x) - 4L + 2m < \varepsilon$$

$$\Rightarrow -\varepsilon < 4f(x) - 2g(x) - (4L - 2m) < \varepsilon$$

$$\Rightarrow |4f(x) - 2g(x) - (4L - 2m)| < \varepsilon \Rightarrow \lim_{x \rightarrow c} (4f(x) - 2g(x)) = 4L - 2m$$

$$\varepsilon > -2g(x) + 2m > -\varepsilon$$

$$\Rightarrow -2\varepsilon - \varepsilon < 4f(x) - 2g(x) - (4L - 2m) < 2\varepsilon + \varepsilon$$