43.

prove $\lim\limits_{x\to 1}\frac{1}{x}=1$

$0<|x-c|<\delta$, $|f(x)-L|<\varepsilon$.

Given $1>\varepsilon>0$, there exists $\delta>0$.

$0<|x-c|<\delta$, $|f(x)-L|<\varepsilon$

$\left|\frac{1}{x}-1\right|<\varepsilon$. (A)

$\Leftarrow -\varepsilon<\frac{1}{x}-1<\varepsilon$.

$\Leftarrow -\varepsilon+1<\frac{1}{x}<\varepsilon+1$

$\Leftarrow \frac{1}{-\varepsilon+1}>x>\frac{1}{\varepsilon+1}$ (B)

$-\delta<x-1<\delta$

$-\delta+1<x<\delta+1$

$$\begin{cases} 1-\delta \geqslant \frac{1}{\varepsilon+1} \\ \delta+1 \leqslant \frac{1}{1-\varepsilon} \end{cases}$$

$$\begin{cases} \delta \leqslant 1-\frac{1}{1+\varepsilon}=\frac{\varepsilon}{1+\varepsilon} \\ \delta \leqslant \frac{1}{1-\varepsilon}-1=\frac{\varepsilon}{1-\varepsilon} \end{cases}$$

$\therefore \delta=\min\left\{\frac{\varepsilon}{1+\varepsilon},\frac{\varepsilon}{1-\varepsilon}\right\}$

$=\frac{\varepsilon}{1+\varepsilon}$

因為要說明所選取的delta可以從(B)

推到(A), 反向的箭頭是必須的