

43.

$$0 < |x-1| < \delta$$

$$\Rightarrow -\delta < x-1 < \delta$$

$$\Rightarrow -\delta+1 < x < \delta+1$$

$$|\frac{1}{x}-1| < \epsilon \quad (A)$$

$$\Leftrightarrow -\epsilon < \frac{1}{x}-1 < \epsilon$$

$$\Leftrightarrow -\epsilon+1 < \frac{1}{x} < \epsilon+1 \quad \text{assume } 0 < \epsilon < 1$$

$$\Leftrightarrow \frac{1}{1+\epsilon} < x < \frac{1}{1-\epsilon} \quad (B)$$

$$\begin{cases} \delta+1 \leq \frac{1}{1-\epsilon} \\ -\delta+1 \geq \frac{1}{1+\epsilon} \end{cases} \Rightarrow \begin{cases} \delta \leq \frac{1}{1-\epsilon} - 1 \\ \delta \leq 1 - \frac{1}{1+\epsilon} \end{cases}$$

$$\text{take } \delta = \min \left\{ \frac{1}{1-\epsilon} - 1, 1 - \frac{1}{1+\epsilon} \right\}$$

因為要說明所選取的delta可以從(B)推到(A),反向箭頭是必要的