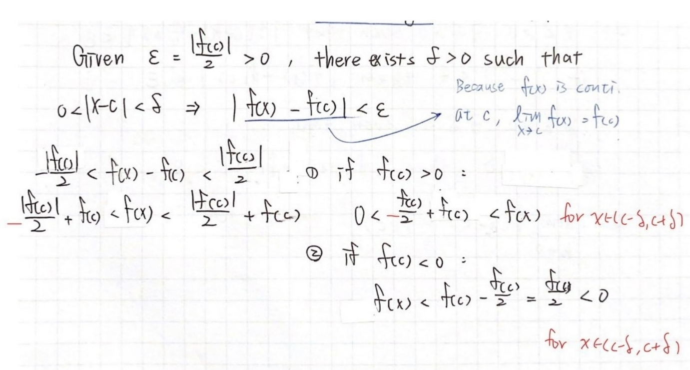
Study guide for quiz 02

Quiz problems include both the lecture contents and homework problems.

1. Practice on variants of $\lim_{\theta \to 0} \dfrac{\sin\theta}{\theta}$.
2. Study the precise definitions of one-sided limit and continuity in terms of ε and δ.
3. Study the proof of Theorem 9, composition of continuous functions (in Lecture 4) and Theorem 10, limits of continuous functions (in textbook).
4. Study the Intermediate Value Theorem and its applications including root locating.

1 Brief answer and some common mistakes in section 2.5, problem 68



Given $\varepsilon = \frac{|f(c)|}{2} > 0$, there exists $\delta > 0$ such that

$0 < |x-c| < \delta \Rightarrow |f(x) - f(c)| < \varepsilon$

Because $f(x)$ is conti. at c, $\lim_{x \to c} f(x) = f(c)$

$-\frac{|f(c)|}{2} < f(x) - f(c) < \frac{|f(c)|}{2}$

$-\frac{|f(c)|}{2} + f(c) < f(x) < \frac{|f(c)|}{2} + f(c)$

① if $f(c) > 0$:

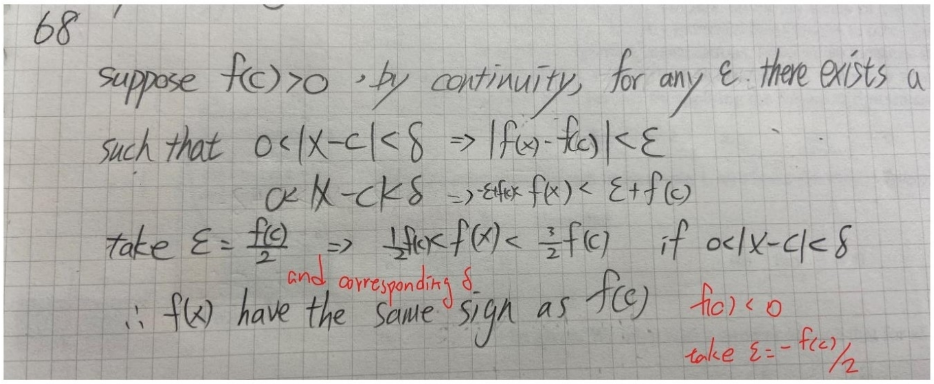
$0 < -\frac{f(c)}{2} + f(c) < f(x)$ for $x \in (c-\delta, c+\delta)$

② if $f(c) < 0$:

$f(x) < f(c) - \frac{f(c)}{2} = \frac{f(c)}{2} < 0$

for $x \in (c-\delta, c+\delta)$

Figure 1: Correct answer section 2.5, problem 68 (only minor mistake)



68

suppose $f(c)>0$, by continuity, for any ε there exists a

such that $0<|x-c|<\delta \Rightarrow |f(x)-f(c)|<\varepsilon$

$0<|x-c|<\delta \Rightarrow -\varepsilon+f(c)<f(x)<\varepsilon+f(c)$

take $\varepsilon=\frac{f(c)}{2}$ and corresponding δ $\Rightarrow \frac{1}{2}f(c)<f(x)<\frac{3}{2}f(c)$ if $0<|x-c|<\delta$

$\therefore$ $f(x)$ have the same sign as $f(c)$

$f(c)<0$

take $\varepsilon=-f(c)/2$

Figure 2: Most common mistake of section 2.5, problem 68